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STATISTICAL, NONLINEAR, AND SOFT MATTER PHYSICS

Anomalous Response of Stratified Fluid to Forcing

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Abstract—The time-independent disturbances induced by nonuniformly distributed surface shear stress in a binary fluid (salt water) are analyzed in the linear approximation. It is shown that correct treatment of two scalar fields (stratified background temperature and salinity distributions) may lead to counterintuitive qualitative predictions even if the thermal and salt diffusivities are equal. In particular, when stable salinity stratification is imposed on stable temperature stratification, both the amplitude and depth of the disturbance may substantially increase rather than decrease (contrary to intuitive expectations). A previously unknown mechanism of convective instability is revealed for a stable density-stratified binary fluid.

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1. INTRODUCTION

The disturbances induced in a fluid by shear stress nonuniformly distributed over a horizontal fluid surface are of great interest for a number of applications. These include some classical problems in geophysics, such as response of the upper layer of a water body to wind forcing [1], and certain mechanisms of convection in unstable two-layer systems discussed in recent years ("anticonvection" [2]). In these systems, convective motion in one layer gives rise to a horizontally nonuniform distribution of shear stress over the interface, which induces flow and temperature disturbances in the other layer that can sustain convection in the former one through positive feedback. One key factor in the dynamics of these systems is response of stratified fluid to shear stress nonuniformly distributed over a horizontal boundary. The present analysis is focused on the response of a binary fluid (such as seawater) stratified with respect to both temperature and salinity. Anomalous response of such fluids to horizontally nonuniform heating was revealed in a previous study [3].

2. STATEMENT OF THE PROBLEM

Figure 1 schematizes the geometry of the two-dimensional problem under analysis. The half-plane $z \le 0$ is occupied by a fluid stratified with respect to both temperature and salinity. While temperature and salinity distributions may be unstably stratified, the overall density stratification is supposed to be stable; i.e., the fluid is in hydrostatic equilibrium.

The linearized time-independent system of equations for perturbations is written in the Boussinesq approximation as follows [4, 5]:

$$0 = -\frac{1}{\rho_0}\nabla p + \nu\nabla^2\mathbf{v} + g(\alpha T - \beta s)\mathbf{e}_z, \quad \nabla\mathbf{v} = 0, \qquad (1)$$
$$\gamma_T\mathbf{v}\cdot\mathbf{e}_z = \lambda\nabla^2 T, \quad \gamma_s\mathbf{v}\cdot\mathbf{e}_z = \chi\nabla^2 s.$$

Here, T, s, $\mathbf{v}$, and p denote temperature, salinity, velocity, and pressure perturbations; $\mathbf{e}_z$ is the unit vector along the z axis; ρ_0 is the reference fluid density; ν is kinematic viscosity; λ is thermal diffusivity; χ is salt diffusivity; α is the thermal expansion coefficient of the fluid; β is the coefficient of salinity compression; and g is the gravitational acceleration. As mentioned above, the background vertical gradient of temperature and salinity, γ_T and γ_s, are supposed to be such that the background stratification is stable [4–6].

Time-independent shear stress at the fluid surface is modeled by a harmonic function of x:

$$\rho_0\nu\frac{\partial u}{\partial z} = E\sin(kx) \text{ at } z = 0, \qquad (2)$$

where E is the stress amplitude, u is horizontal velocity,

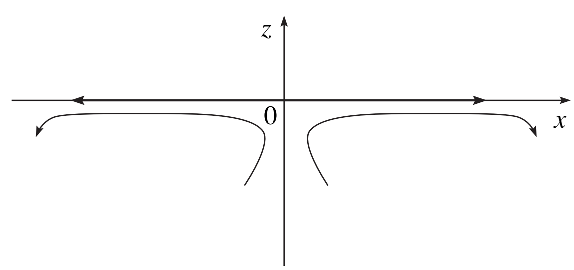

Fig. 1. Schematic of the problem geometry. Arrows represent nonuniformly distributed surface shear stress and streamlines of the induced flow.

and k is the wavenumber. Surface distortion is neglected; i.e., the vertical velocity w vanishes at $z = 0$. Boundary conditions of the third kind for temperature and salinity are set on the surface:

$$\frac{\partial T}{\partial z} = -\frac{T}{h_T}, \quad \frac{\partial s}{\partial z} = -\frac{s}{h_s} \text{ at } z = 0, \tag{3}$$

where h_T and h_s are length scales. All perturbations are assumed to decay away from the surface (as $z \longrightarrow -\infty$).

3. PRELIMINARY ESTIMATES

A nonuniform distribution of surface shear stress induces a horizontally nonuniform flow in the fluid. If u is a function of x, then vertical fluid motion must develop by virtue of flow continuity, giving rise to heat and salinity fluxes in a stratified fluid. The yet unknown depth of time-independent disturbance in a stably stratified medium is denoted by H. Boundary condition (2) can be used to estimate the velocity amplitude for the induced horizontal flow:

$$\frac{\rho_0 \nu u}{H} \sim E, \quad u \sim \frac{EH}{\rho_0 \nu} \tag{4}$$

(the velocity perturbation amplitude is also denoted by u for simplicity). From the continuity equation, it follows that

$$ku \sim \frac{w}{H}, \quad w \sim Hku \sim \frac{EH^2 k}{\rho_0 \nu}. \tag{5}$$

The heat equation can be used to estimate the amplitude of the induced temperature perturbation:

$$\gamma_T w \sim \frac{\lambda T}{H^2}, \quad T \sim \frac{\gamma_T w H^2}{\lambda} \sim \frac{\gamma_T k E H^4}{\rho_0 \lambda \nu}. \tag{6}$$

Here, only the vertical component of the Laplace operator in the heat equation is taken into account, because it can be shown that the vertical length scale H of perturbations (disturbance depth) cannot exceed their horizontal scale $L = 2\pi/k$ in order of magnitude. For the same reason, the hydrostatic approximation can be used in preliminary estimates:

$$p' \sim g\rho' H \sim g\rho_0 |\alpha T - \beta s| H, \tag{7}$$

where the primes denote the pressure and density perturbation amplitudes. Furthermore, the essential terms in the horizontal component of the equation of motion are assumed to be comparable in order of magnitude:

$$\nu \frac{\partial^2 u}{\partial z^2} \sim \frac{\nu u}{H^2} \text{ and } \frac{1}{\rho_0}\frac{\partial p'}{\partial x} \sim g|\alpha T - \beta s| kH.$$

Hence,

$$|\alpha T - \beta s| \sim \frac{E}{\rho_0 g k H^2}. \tag{8}$$

In the case of a one-component medium (when s is ignored), (6) and (8) are combined to obtain estimates for the disturbance depth H and the amplitude T:

$$H \sim \left(\frac{\nu\lambda}{\alpha g \gamma_T k^2}\right)^{1/6}, \quad T \sim \frac{E}{\rho_0 \alpha g k H^2} \sim \frac{E}{\rho_0}\left(\frac{\gamma_T}{\nu \lambda k (\alpha g)^2}\right)^{1/3}. \tag{9}$$

The solution of the problem for a one-component medium stratified only with respect to temperature leads to this result [1], which is predicted here by a simple scale analysis. The vertical length scale H given by (9) has long since been known. (In Western literature, it has been called the Lineykin scale, after a prominent Russian oceanologist [7]). It has been interpreted as the depth of the disturbance caused in a stably stratified medium by time-independent surface forcing of different nature in [1, 7] and depends on the buoyancy (Brunt–Vaĭsälä) frequency $N_T = (\alpha g \gamma_T)^{1/2}$. When the salinity stratification is also taken into account, the buoyancy frequency must be different. At first glance, it would be sufficient to introduce an appropriate correction to the expression for the frequency (or, equivalently, to the density stratification). Since the value of H given by (9) weakly depends on stratification ($H \sim N_T^{-1/3}$), it might be expected that allowance for salinity stratification would result only in some quantitative corrections. In what follows, it is shown that qualitatively different results are obtained for a binary fluid. The thermal response to mechanical forcing may strongly deviate from the seemingly obvious result of the scale analysis.

4. SOLUTION

Time-independent solutions are sought as harmonic functions of x:

$$u(x, z) = U(z)\sin(kx), \quad w(x, z) = W(z)\cos(kx),$$
$$T(x, z) = \theta(z)\cos(kx), \quad s(x, z) = \vartheta(z)\cos(kx),$$

etc. By eliminating all unknowns except w from (1), the following equation is obtained:

$$\left(\frac{d^2}{dz^2} - k^2\right)^3 W = k^6 S W, \tag{10}$$

where

$$S = \frac{1}{k^4\nu}\left(\frac{N_T^2}{\lambda} + \frac{N_s^2}{\chi}\right) = R_T + \frac{R_s}{\delta},$$

$N_T^2 = \alpha g \gamma_T$ and $N_s^2 = -\beta g \gamma_s$ are the temperature and salinity contributions to the squared buoyancy frequency $N^2 = N_T^2 + N_s^2$, respectively;

$$R_T = \frac{N_T^2}{\nu\lambda k^4}, \quad R_s = \frac{N_s^2}{\nu\lambda k^4}$$

are analogs of the Rayleigh number; and $\delta \equiv \chi/\lambda$.

The dimensional parameter S is interpreted as a generalized Rayleigh number, which is nonnegative for the domain of admissible values of γ_T and γ_s considered here.

The solution to Eq. (10) is sought in the form of a linear combination of the exponentials $\exp(q_i kz)$, where q_i are the roots of the characteristic equation

$$(q^2 - 1)^3 = S. \tag{11}$$

Since only three of the six exponentials decay as $z \longrightarrow -\infty$, the coefficients of the remaining ones should be set to zero, and the vertical velocity can be represented as

$$W(z) = \sum_{i=1}^{3} C_i \exp(q_i kz), \tag{12}$$

$$q_1 = (1 + S^{1/3})^{1/2},$$
$$q_{2,3} = \left[1 + S^{1/3}\exp\left(\pm\frac{2}{3}\pi i\right)\right]^{1/2}, \tag{13}$$

where C_i are integration constants and $\mathrm{Re}\, q_i > 0$. The analysis of solution presented below is restricted to the case of $S^{1/6} \gg 1$, when the flow pattern developing in the fluid is primarily determined by stable density stratification. Under this condition, the solution can be considerably simplified by dropping unity in both (11) and (13):

$$q_1 \approx S^{1/6}, \quad q_{2,3} \approx S^{1/6}\exp\left(\pm\frac{1}{3}\pi i\right).$$

Since the exponents in (12) and in the expressions for u, as well as in those for pressure and buoyancy perturbations, contain the large factor $S^{1/6}$, the corresponding disturbance depth is small as compared to the horizontal length scale $L = 2\pi/k$. In other words, the aspect ratio of perturbations defined as the horizontal-to-vertical length-scale ratio is much greater than unity. Therefore, the x component of the Laplace operator in Eq. (1) can be neglected, and the characteristic vertical velocity is much lower than the horizontal one. Moreover, a scale analysis shows that the entire viscous term in the vertical component of Eq. (1) can be dropped; i.e., the model can be reduced to the hydrostatic approximation, as commonly done when flow disturbances are stretched in the horizontal direction. Thus, the components of the equation of motion become

$$0 = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2},$$

$$0 = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} + g(\alpha T - \beta s).$$

(These simplifications are used to obtain the estimates presented above.)

A simple analysis of the equations for temperature and salinity perturbations in (1) shows that the general expressions for temperature and salinity contain additional terms proportional to $\exp(kz)$ as compared to (12):

$$T = \frac{\gamma_T}{\lambda k^2}\left(C'_T \exp(kz) + \sum_{i=1}^{3}\frac{C_i}{q_i^2 - 1}\exp(kq_i z)\right)\cos(kx),$$

$$s = \frac{\gamma_s}{\chi k^2}\left(C'_s \exp(kz) + \sum_{i=1}^{3}\frac{C_i}{q_i^2 - 1}\exp(kq_i z)\right)\cos(kx).$$

By substituting these expressions into the remaining equations in (1), it is found that the integration constants C'_T and C'_s are related as follows:

$$C'_s = -\frac{\chi N_T^2}{\lambda N_s^2}C'_T. \tag{14}$$

This relation must hold to ensure that the expression for the buoyancy $g(\alpha T - \beta s)$ does not contain the factor $\exp(kz)$. Otherwise, this exponential will also be contained in the expression for **v** by virtue of Eq. (1), which contradicts (12). The difference between the temperature and salinity fields, on one hand, and the density, velocity, and pressure fields, on the other, is an essential distinction of the present solution as compared to that for a one-component medium (e.g., see [1]).

The integration constants are calculated by using the boundary conditions. The solution is

$$u = -S^{1/6}e^{Kz}\left[C_1 e^{Kz} - 2\left(C_1 + \frac{\sqrt{3}}{4}\tau\right)\right.$$
$$\left.\times\cos(\sqrt{3}Kz) - \frac{\tau}{2}\sin(\sqrt{3}Kz)\right]\sin(kx),$$

$$w = e^{Kz}[C_1(e^{Kz} - \cos(\sqrt{3}Kz)) - (\sqrt{3}C_1 + \tau)\sin(\sqrt{3}Kz)]\cos(kx),$$

$$T = \frac{\gamma_T}{\lambda k^2 S^{1/3}} \times \left\{ C_T e^{kz} + e^{Kz}\left[C_1 e^{Kz} + 2\left(C_1 + \frac{\sqrt{3}}{4}\tau\right) \times \cos(\sqrt{3}Kz) + \frac{\tau}{2}\sin(\sqrt{3}Kz)\right]\right\}\sin(kx),$$

$$s = \frac{\gamma_s}{\chi k^2 S^{1/3}} \times \left\{ \frac{\chi\alpha\gamma_T}{\lambda\beta\gamma_s} C_T e^{kz} + e^{Kz}\left[C_1 e^{Kz} + 2\left(C_1 + \frac{\sqrt{3}}{4}\tau\right) \times \cos(\sqrt{3}Kz) + \frac{\tau}{2}\sin(\sqrt{3}Kz)\right]\right\}\sin(kx),$$

where

$$K = \frac{kS^{1/6}}{2}, \quad \tau = \frac{Ek}{2\sqrt{3}K^2\nu\rho_0} = \frac{2E}{\sqrt{3}k\nu\rho_0 S^{1/3}},$$

$$C_T = S^{1/3}C'_T = -\left(\frac{h_T}{h_s} - 1\right)\Phi, \tag{15}$$

$$C_1 = -\left[(1 + kh_T)\left(1 + \frac{1}{2Kh_s}\right) - \frac{\chi\alpha\gamma_T h_T}{\lambda\beta\gamma_s h_s}(1 + kh_s)\left(1 + \frac{1}{2Kh_T}\right)\right]\Phi,$$

$$C_1 + \frac{\sqrt{3}}{4}\tau = \frac{1}{4Kh_s}\left[1 + kh_T - \frac{\chi\alpha\gamma_T}{\lambda\beta\gamma_s}(1 + kh_s)\right]\Phi,$$

$$\Phi = \frac{\sqrt{3}}{4}\tau\left[(1 + kh_T)\left(1 + \frac{3}{4Kh_s}\right) - \frac{\chi\alpha\gamma_T h_T}{\lambda\beta\gamma_s h_s}(1 + kh_s)\left(1 + \frac{3}{2Kh_T}\right)\right]^{-1}. \tag{16}$$

5. ANALYSIS

The solution depends on the dimensionless parameters S, $N_s^2/N_T^2 = R_s/R_T$, $\delta = \chi/\lambda$, kh_T, and kh_s (the last two are inversely proportional to the corresponding analogs of the Biot number [8]). Distinctive fluid-dynamic and thermodynamic characteristics of binary fluids are generally attributed to the difference between the diffusivities χ and λ [4, 5]. An analysis of the solution obtained here shows that nontrivial effects are possible when $\delta = 1$ as well.

The expressions for the velocity components and the pressure and density perturbations are linear combinations of three functions: $\exp(2Kz)$, $\exp(Kz)\cos(\sqrt{3}Kz)$, and $\exp(Kz)\sin(\sqrt{3}Kz)$. These exponentials decay over a vertical length scale $H \sim L/S^{1/6}$, which is much smaller than $L \equiv 2\pi/k$, and the corresponding wavelengths are comparable to H. This result is analogous to the solutions obtained for one-component media [1, 7], which describe vertically arranged circulation cells having vertical and horizontal dimensions on the order of H and L, respectively, with circulation intensity decaying over a similar vertical length scale H.

Whereas the above simplified analysis of velocity, pressure, and density perturbations in horizontally stretched near-surface circulation cells is valid for small H/L, temperature perturbations in a binary fluid exhibit a qualitatively different behavior due to the slowly decaying term $\exp(kz)$ in the expressions for temperature and salinity perturbations.

At first glance, salinity stratification can only modify density stratification and the effective Rayleigh number (if the double-diffusive convection due to difference between diffusivities is ignored). For this reason, problems of this kind are frequently simplified as follows [1]. Linear combination of the heat and salinity equations with equal diffusivities is used to derive an equation for density (or buoyancy) allowing for the overall background density stratification. It is frequently supposed that this reduces the analysis to the known solution for a one-component medium. However, a consistent boundary condition for density at $z = 0$ cannot be formulated, because the two variables contributing to density perturbations (temperature and salinity) are subject to quantitatively different boundary conditions in the general case. The solution written out above demonstrates that a qualitatively different behavior is predicted without relying on the poorly justified simplification mentioned above. The slowly decaying term $\exp(kz)$ in the expressions for temperature and salinity implies that disturbances of these variables can be much deeper in binary fluids as compared to one-component ones (to depths on the order of L), other parameters being equal. Coefficient (15) of this exponential vanishes for a one-component fluid, as well as in the case when $h_T = h_s$. The fact that buoyancy, pressure, and velocity in binary fluids are disturbed only to depths on the order of H (as in one-component media) is explained by the mutually compensating effects of temperature and salinity perturbations.

When stable salinity stratification is imposed on stable temperature stratification, it may seem obvious that an increase in stability of stratification must restrain the disturbance of surface disturbances into the medium. However, the solution presented above shows that, con-

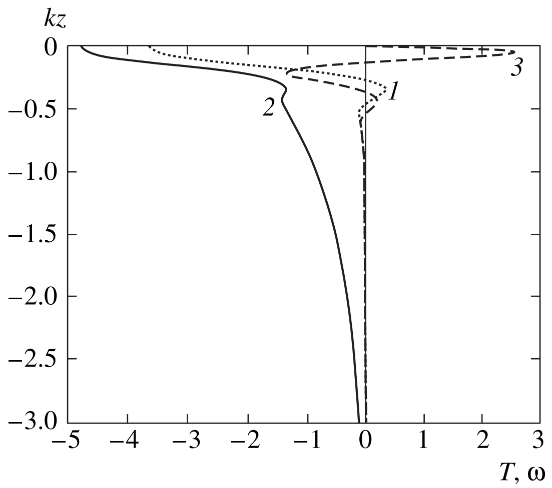

Fig. 2. Profiles of temperature disturbances at $x = 0$ (normalized to $\sqrt{3} \times 10^{-3}\gamma_T\tau/4\lambda k^2$) for the numerical values of parameters specified in text, $\chi\alpha\gamma_T/\lambda\beta\gamma_s = -1$, $kh_T = 1$, and $kh_s = 0.05$. Curves *1* and *2* correspond to one-component and binary fluids. Curve *3* is the vertical velocity profile in a binary fluid normalized to $2\sqrt{3}\,\tau$.

trary to intuitive expectations, the characteristic temperature disturbance depth increases from H to L.

An increase in stable stratification corresponds to a larger effective Rayleigh number S and an ensuing faster decay of velocity, density, and pressure perturbations containing the factor $\exp(Kz)$. However, even the increase in $S^{1/3}$ in the denominators of the expressions for temperature and salinity cannot compensate for the substantial increase in the depth of temperature disturbance in a binary fluid described by the additional exponential $\exp(kz)$.

As an example amenable to laboratory experiments, consider first a one-component fluid (water) stratified only with respect to temperature, with $L = 0.5$ m, $k = 2\pi/L \approx 12.5$ m^{-1}, $\gamma_T = 30$ K/m, $\nu = 10^{-6}$ m^2/s, $\lambda = 1.4 \times 10^{-7}$ m^2/s, and $\alpha = 2 \times 10^{-4}$ K^{-1}. In this case, $N_T \sim 0.25$ s^{-1} and $\mathrm{Ra}_T = S \sim 1.5 \times 10^7$. Figure 2 compares the vertical profiles of temperature and vertical velocity for this medium with those calculated for stable salinity stratification (with $\gamma_s < 0$) imposed on stable temperature stratification, when the effective Rayleigh number is $S = 3 \times 10^7$ (twice as high).

The increase in S by a factor of two corresponds to a relatively small increase in $S^{1/3}$ or $S^{1/6}$ i.e., the ensuing changes in the velocity, density, and pressure fields are insignificant (for this reason, Fig. 2 shows only one profile of vertical velocity). However, the temperature profiles shown in Fig. 2 demonstrate that the qualitatively different temperature profile predicted for a binary fluid is even more pronounced in the case of more stable stratification.

The foregoing discussion was focused on the case when both scalar fields are stably stratified ($\gamma_T > 0$, $\gamma_s < 0$, $\gamma_T/\gamma_s < 0$). If one of the fields is unstably stratified ($\gamma_T/\gamma_s > 0$), then the perturbation amplitude increases indefinitely as the expression in brackets in (16) vanishes, which implies unstable background state. The convective instability of a binary fluid for which $\chi\alpha\gamma_T/\lambda\beta\gamma_s < 1$ (double-diffusive instability) is a well studied phenomenon [4–6]. However, the much less restrictive condition for instability depending on h_T and h_s derived at the end of the next section points to a new type of instability.

6. PHYSICAL MECHANISM OF STRONG RESPONSE

6.1. Stable Stratification of Both Scalar Fields

The large disparity between curves *1* and *2* is due to a seemingly insignificant difference in background density stratification. When $\chi/\lambda = 10^{-2}$ (as in sea water), one would expect that weak salinity stratification imposed on stable temperature stratification should strengthen the background stable stratification and result in a relatively small decrease in both amplitude and depth of temperature disturbance induced by surface shear stress. However, the actual effect is totally different: in the particular numerical example considered above, the temperature disturbance depth increases by approximately an order of magnitude.

To be specific, consider the case of when the shear stress in the neighborhood of $x = 0$ has a positive divergence, as illustrated by Figs. 1 and 2. By flow continuity, the induced divergent horizontal flow gives rise to upwelling (see Fig. 1 and curve *3* in Fig. 2), which results in a lower temperature and a higher density of the surface layer (curves *1* and *2* in Fig. 2). For a one-component fluid, the amplitude and depth of temperature disturbance are given by estimates (9) derived from balance between the forced upwelling and the negative buoyancy of the relatively cooler and denser ascending fluid parcels.

In a binary fluid, the buoyancy of a fluid parcel depends not only on its temperature, but also on salinity. In the case considered here, their respective effects on the buoyancy field are mutually compensating: an “anomalously cool” surface layer described by the solution presented above must be “anomalously fresh”; i.e., the balance between forcing and change in buoyancy is maintained even though estimates (9) do not hold. The difference in boundary conditions between the scalar fields ($h_s \neq h_T$) has a separating effect on their distributions, which manifests itself in anomalously strong temperature disturbances.

In any fluid, buoyancy forces ensure that isopycnal surfaces are flat everywhere except for a thin surface layer of thickness H, where this alignment is distorted by forcing. In a one-component medium with density determined by temperature, isotherms are aligned correspondingly; i.e., temperature disturbances are localized in the same thin layer. In a binary fluid, where the scalar fields can be separated, the flatness of isopycnals does not preclude the development of strong perturbations of either field, as in the example considered above.

6.2. Unstable Salinity Stratification

When $\gamma_T/\gamma_s > 0$, the expression in brackets in (16) may vanish (see above). An indefinitely strong response implies that the background distribution becomes unstable, and the corresponding condition is

$$\frac{\chi\alpha\gamma_T}{\lambda\beta\gamma_s} < \frac{(1+kh_T)(\varepsilon+kh_s)}{(1+kh_s)(\varepsilon+kh_T)} = \frac{1+kh_T}{\varepsilon+kh_T}\frac{\varepsilon+kh_s}{1+kh_s}, \quad (17)$$

where $\varepsilon = 3/2S^{1/6}$ is a small parameter. When $h_T = h_s$, the right-hand side in (17) is unity, which is equivalent to a well-known condition for double-diffusive instability [4–6]. In the case of different boundary conditions for temperature and salinity, condition (17) is much less restrictive. The first fraction on the right-hand side of (17) depends only on h_T and reaches a maximum (ε^{-1}) when $h_T = 0$. The other fraction depends only on h_s, monotonically increasing with this parameter (approaching unity). Therefore, the right-hand side in (17) reaches a maximum (ε^{-1}) when $h_T = 0$ and $h_s \longrightarrow \infty$.

The foregoing analysis points to a new type of instability in hydrostatically stable binary fluids. It is generally believed that a fluid in this state can become unstable only when $\chi \ll \lambda$. However, since the condition $\chi\alpha\gamma_T/\lambda\beta\gamma_s < 1$ is replaced by the much less restrictive

$$\frac{\chi\alpha\gamma_T}{\lambda\beta\gamma_s} < \varepsilon^{-1} \equiv \frac{2S^{1/6}}{3},$$

a stable density-stratified fluid may become unstable even in the case of equal diffusivities when a weakly unstable salinity stratification (with $\gamma_s \ll \gamma_T$) is imposed. The physical mechanism underlying the instability can be explained as follows. Consider the case of stable temperature stratification and unstable salinity stratification. Since the density stratification is stable, a fluid parcel displaced upwards experiences a restoring force as its buoyancy grows negative. However, the local buoyancy depends on exchange with the ambient fluid. If the temperature at the surface is less prone to variation than salinity ($h_T < h_s$), then temperature relaxation in the fluid parcel is faster than salinity relaxation when other parameters are equal. Therefore, the buoyancy may not become negative and may even increase as the local salinity decreases; i.e., the restoring force may not be generated. This mechanism is somewhat analogous to double-diffusive instability [4, 5], but the explanation lies in the difference between boundary conditions rather than diffusivities. Instability of this type is most likely to develop when $h_T = 0$ and $h_s \longrightarrow \infty$ (temperature and salinity are subject to boundary conditions of the first and second kind, respectively).

Note that a mechanism of instability due to difference between boundary conditions for different scalar fields was suggested in [9]. However, the analysis presented therein was performed for other combinations of stratified temperature and salinity distributions, a substantially different mechanism was considered, and an oscillatory instability was postulated. An oversimplified model was analyzed (in particular, viscous effects were neglected), and the stability problem remained unsolved.

7. CONCLUSIONS

Some previously known “striking” [4] fluid-dynamic and thermodynamic characteristics of binary mixtures have been explained by double-diffusive convection due to difference in diffusivity between scalar fields (differential diffusion) [4]. By contrast, the strong response of temperature field to forcing described above can be observed when the diffusivities are equal and both fields are stably stratified. It is shown that if the contribution of one of the fields to density stratification has a slightly destabilizing effect, then the onset of instability may be due to the difference between boundary conditions, rather than the difference in diffusivity.

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