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On the Effect of Spray on the Dynamics of the Marine Atmospheric Surface Layer in Strong Winds

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Abstract—A nonlinear analytical model of the near-water air layer is considered. In the model, the effect of spray droplets on turbulent exchange is taken into account through their effect on density stratification. To close the system of equations, a semiempirical turbulence theory in the Kolmogorov–Monin form is used. Unlike previous publications, we use an alternative closure scheme that seems more appropriate. A version is also proposed for generalizing the model to the case where heavy admixture particles (spray droplets) make a dominant contribution to the average density of a medium. This model allows a general analytical solution that, in principle, describes nonlinear effects such as a decrease in the effective friction and “self-closure” for the heavy admixture in the near-surface layer because of turbulence suppression due to the strengthening of stratification stability. As the current data show, however, the intensity of spray production is probably insufficient to explain the recently discovered phenomenon of the aerodynamic-drag decrease (saturation) in storm winds.

Keywords: near-water layer, storm conditions, spray, turbulence suppression, aerodynamic drag, nonlinear models

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INTRODUCTION

Atmosphere–ocean circulation has long been thought to be enhanced in strong winds [1–3]. Recent field data and the results of laboratory experiments, however, uncovered a nontrivial phenomenon: the aerodynamic drag coefficient decreases (saturates) in storm winds (the effect of drag crisis) [4–8]. This is quite important for the dynamics of tropical cyclones and polar mesolows. One mechanism behind this phenomenon is the effect of spray droplets on the structure and dynamics of the marine atmospheric layer [6–10]. When spray production intensifies, the effective density stratification in the lower air becomes more stable, which in principle leads to a decrease in turbulence and effective viscosity in this layer. A priori it is not improbable that this can reduce the influence that wind has on the sea surface, with other conditions being equal. Such a mechanism was proposed in [6] and analyzed in detail in [7]. Similar ideas were developed separately in [9, 10].

The results that have been obtained until now are ambiguous. In [7] it is concluded that suspended spray (spray droplets less than or on the order of 20 μm in radius) cannot significantly influence the aerodynamic drag. According to this study, the drag could in principle be affected by larger spray droplets (about 180 μm in radius) torn off from wave crests by the wind. Estimates show, however, that the levitation of such droplets requires unrealistically high wind speeds. In [9, 10], on the contrary, it is concluded that turbulent viscosity can significantly decrease due to the spray-induced stable density stratification.

The uncertainty in the results occurs largely because the scatter of experimental data on the amount of spray droplets in storm winds is very large [7, 11–13]. Also, the description of turbulent exchange in studies that we know of is not accurate in every respect.

This applies mainly to the commonly accepted hypothesis on the turbulence scale (e.g., formula (11) in [7]). The turbulence scale is hypothesized to be proportional to the vertical coordinate z (the distance from the underlying surface). We think that such a hypothesis is unlikely to be justified in a strong stable stratification (the most interesting situation in a given context). From physical considerations, it is more correct to assume that the turbulence scale decreases as the hydrostatic stability increases, so turbulent fluctuations away from the underlying surface cease to interact with the surface and cease to depend on the distance z. This agrees with considerations given in a number of works and outlined, for example, in [14] (p. 318): “The existence of large eddies becomes impossible in the case of stable stratification (since they must spend too much energy on opposing the buoyancy force) and turbulence can exist only in the form of small eddies. In such a situation, turbulent exchange between different atmospheric layers is

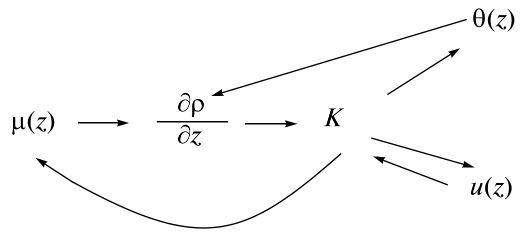

Fig. 1. Scheme of the main connections in the model. $\mu(z)$ is the partial density of the heavy admixture (spray droplets), ρ is the air density, and K is the coefficient of turbulent viscosity.

hampered; turbulence becomes local; *the wall no longer affects the turbulence regime; and, hence, the turbulence coefficient should not depend on z"* (emphasis mine). Of course, it is implied here that the turbulence coefficient is independent of the closeness of the wall; if local conditions (stratification or wind shear) depend on z, the dependence of the turbulence coefficient K on z related to these factors is not improbable. In this paper, therefore, an analogous problem is considered using an alternative turbulence closure scheme, which in our case seems to be more appropriate.

In some of the papers mentioned above, the parameter u_* (friction velocity) is specified. In our paper, this parameter is found from a solution of the problem, which seems more logical. Furthermore, we know about publications that are confined to the case of the small contribution that spray makes to the average air density. Indeed, the semiempirical turbulence theory used in the atmospheric models contains a simplification that is usually beyond question. In the cases we consider, the justification of this simplification is not as obvious. Therefore, in our paper we also consider a generalized version of the model with an additional hypothesis for when spray makes the dominant contribution to the average air density.

We note that the possible influence that spray has on the near-water density stratification has long been discussed (see, e.g., [3]). This effect was usually associated with the cooling of air by spray evaporation. Such an effect should be really taken into account, but at high wind speeds the influence that cooling has on the effective Richardson number Ri_θ seems small. We now make an estimate of this influence.

$$\mathrm{Ri}_\theta = \frac{\alpha g(\partial\theta/\partial z)}{(\partial u/\partial z)^2} \sim \frac{\alpha g \Delta\theta H}{u^2},$$

where the axis z is directed vertically upward, u is the horizontal velocity, θ is the potential temperature of air, α is its thermal expansion coefficient, g is the gravity acceleration, H is the layer thickness, and $\Delta\theta$ is the vertical difference of the potential temperature. If the cooling due to the evaporation of droplets is 3 K, then at $H = 30$ m and $u = 30$ m/s this cooling changes Ri_θ by a value on the order of 3×10^{-3}; when $u = 10$ m/s, this value is 3×10^{-2}. Thus, even quite a substantial cooling of the air does not alter the vertical density gradient too much to significantly influence the turbulence regime. In this context, it is interesting to examine the possible role of near-surface water content variations.

2. FORMULATION OF THE PROBLEM

The scheme of connections that a theoretical model is designed to describe is shown in Fig. 1. The heavy admixture (spray droplets) contributes to the density stratification $\partial\rho/\partial z$. The latter affects turbulent exchange. Turbulence determines the transfer of heat and momentum and has an inverse effect on the vertical distribution of spray droplets.

Deviations of the water surface from the horizontal will be ignored. Some reasons for using this essential simplification are thoroughly discussed in [7].

In the approximation of a stationary and horizontally homogeneous near-surface layer (a layer of constant fluxes), the transfer equations of momentum and heat are written in standard form [14–16]

$$K\frac{du}{dz} = u_*^2, \quad \alpha_T K\frac{d\theta}{dz} = Q. \tag{1}$$

Here, α_T is a dimensionless constant (the inverse turbulent Prandtl number), u_* is the friction velocity, and Q is the normalized vertical heat flux (the last two quantities in this formulation of the problem are a priori unknown and are constants of the integration of a one-dimensional Laplace equation). The transfer equation of the spray droplets as a heavy admixture is

$$-w\frac{d\mu}{dz} = \frac{d}{dz}\alpha_\mu K\frac{d\mu}{dz}, \tag{2}$$

where α_μ is a dimensionless constant and w is the absolute value of the gravitational spray sedimentation rate. This assumes significant simplifications, in particular, the neglect of the difference in spray sizes (such a simplification is also used in [7, 9, 10]). Integrating (2) yields

$$\alpha_\mu K\frac{d\mu}{dz} + w\mu = 0 \tag{3}$$

(in selecting an integration constant, the conditions of μ and $d\mu/dz$ damping with height are taken into account).

To describe turbulent transfer, we use the Kolmogorov–Monin balance equation of turbulent energy while taking into account the contribution that the heavy admixture makes to the stratification of a medium [14–19]

$$K\left(\frac{du}{dz}\right)^2 - \alpha_T\alpha g K\frac{d\theta}{dz} + \frac{\alpha_\mu g K}{\bar\rho}\frac{d\mu}{dz} - K^3/c^4l^4 = 0, \tag{4}$$
$$K = lb^{1/2}.$$

Here, $\bar{\rho}$ is the average air density without the contribution of spray, l is the turbulence scale, b is the specific kinetic energy of turbulent fluctuations, and c is a dimensionless constant. In the Boussinesq approximation, the contribution that temperature makes to density stratification is described by $d\rho/dz = -\bar{\rho}\,\alpha d\theta/dz$. Compared to the standard form of the balance equation of turbulent energy, the term with $d\mu/dz$ is added to describe the contribution that the heavy admixture makes to stable density stratification and turbulence suppression (see (6) below). In the balance equation of turbulent energy, the diffusion term is neglected. This is justified in stable stratification, when the turbulence scale is sufficiently small, in particular, when it is much less than characteristic scales of the vertical heterogeneities of average (nonturbulent) profiles. The validity of this simplification can be verified a posteriori, after the solution has been found.

To close the system of (1), (3), and (4), we assume the hypothesis

$$l = sb^{1/2}/N, \tag{5}$$

where

$$N = \left(-\frac{g}{\bar{\rho}}\frac{d\rho}{dz}\right)^{1/2} = \left(\alpha g\frac{d\theta}{dz} - \frac{g}{\bar{\rho}}\frac{d\mu}{dz}\right)^{1/2} \tag{6}$$

is the buoyancy frequency and s is a dimensionless constant. This hypothesis has a transparent physical meaning [20]: the right-hand side of (5), which is accurate to a dimensionless factor, is the distance that the particle of a medium travels vertically with a starting turbulent velocity $b^{1/2}$ before this particle will be stopped by buoyancy forces. It is clear that the turbulence scale cannot exceed the value of the right-hand side of (5). When this value is small (in rather stable stratification), it is also beyond reason to take l much less than (5) (except in a thin near-surface layer $0 < z < l$). This hypothesis agrees with dimension and similarity considerations [14, ch. 11.2; 21]. Hypothesis (5) fundamentally distinguishes this formulation of the problem from [7] and other previous studies in which the transfer of the heavy admixture was modeled. It was assumed there that $1 \sim z$; the turbulence scale, for example, could be greater than the maximum permissible expression (5) with buoyancy forces taken into account and was explicitly dependent on the distance to the underlying surface, contrary to the physical considerations presented above.

Away from the surface $z = 0$, the concentration of spray droplets must tend to zero. At the surface, the concentration of spray droplets, temperature, and no-slip condition are set:

$$u = 0, \quad \theta = \theta_0, \quad \mu = \mu_0 \quad \text{при} \quad z = 0.$$

Other boundary conditions will be detailed below.

3. SOLUTION

Although the system of Eqs. (1), (3)–(6) is essentially nonlinear, it admits a general analytical solution. This solution was found in our paper [22], but the question about the influence that the heavy admixture has on the mechanical water–air interaction was not posed or investigated there (the parameter u_* was specified).

Eliminating all unknowns except μ from the system of equations, we come to a nonlinear equation with separable variables

$$\frac{d\mu}{dz} + \frac{\lambda_\mu g w^2}{\alpha_\mu \bar{\rho} u_*^4}\mu^2 + \frac{\lambda_T w \alpha g Q}{\alpha_\mu u_*^4}\mu = 0, \tag{7}$$

where $\lambda_T = 1 + (s^2c^4\alpha_T)^{-1}$, $\lambda_\mu = 1 + (s^2c^4\alpha_\mu)^{-1}$. Integrating (7), we obtain

$$\begin{aligned} \frac{\mu}{\mu_0} &= \frac{e^{-\xi}}{[1 + M(1 - e^{-\xi})]}, \quad K = (u_*^4/\lambda_T\alpha g Q)f(\xi), \\ \frac{du}{dz} &= (\lambda_T\alpha g Q/u_*^2)[f(\xi)]^{-1}, \\ \frac{d\theta}{dz} &= (\lambda_T\alpha g Q^2/\alpha_T u_*^4)[f(\xi)]^{-1}, \\ u &= \frac{\alpha_\mu u_*^2}{w}\{\xi + \ln[1 + M(1 - e^{-\xi})]\}, \\ \theta - \theta_0 &= \frac{\alpha_\mu Q}{\alpha_T w}\{\xi + \ln[1 + M(1 - e^{-\xi})]\}. \end{aligned} \tag{8}$$

Here,

$$\xi = \frac{z}{h}, \quad h = \frac{\alpha_\mu u_*^4}{\lambda_T \alpha g w Q}, \quad M = \frac{\lambda_\mu \mu_0 w}{\lambda_T \bar{\rho}\alpha Q},$$

$$f(\xi) = \frac{[1 + M(1 - e^{-\xi})]}{(1 + M)}.$$

In the general case, the expressions for N, b, and l are more cumbersome. With an additional assumption $\alpha_\mu\lambda_\mu = \alpha_T\lambda_T$, they have a simple form

$$N = (\lambda_T/\alpha_T)^{1/2}(\alpha g Q/u_*^2)[f(\xi)]^{-1},$$

$$b = u_*^2/s(\alpha_T\lambda_T)^{1/2},$$

$$l = (\alpha_T s^2/\lambda_T^3)^{1/4}(u_*^3/\alpha g Q)f(\xi)^2.$$

Examples of the profiles $\mu(\chi)/\mu_0$ and $f(\chi)$ (the latter coincides with the dimensionless vertical profiles of K and l) for different values of the dimensionless parameter M are given in Fig. 2.

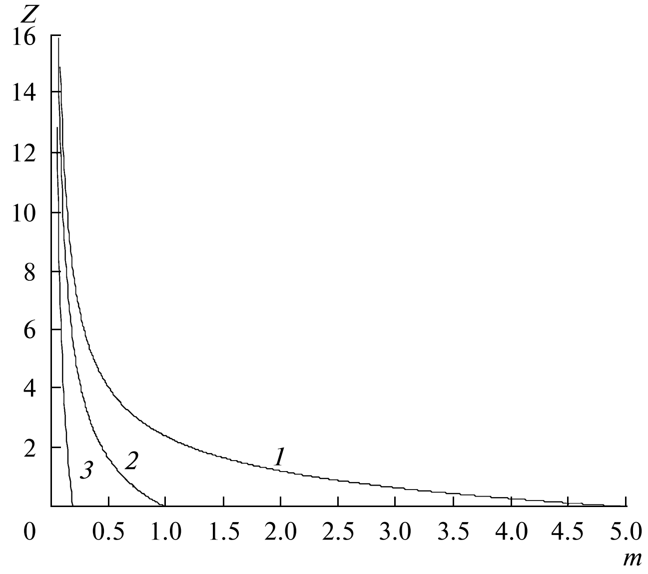

Fig. 2. Dimensionless vertical profiles of the droplet concentration (solution (20)) for $m_0 = 5, 1, 0.2$ (curves *1, 2,* and *3*, respectively).

Also consider expressions for the vertical gradient and spray flux, because there is some information about fluxes in the literature:

$$\frac{d\mu}{dz} = -\frac{\mu_0}{h}\frac{(1+M)e^{-\xi}}{[1+M(1-e^{-\xi})]^2},$$

$$K\frac{d\mu}{dz} = -\frac{\mu_0 w}{\alpha_\mu}\frac{e^{-\xi}}{[1+M(1-e^{-\xi})]} = -\frac{w\mu(z)}{\alpha_\mu}.$$

The surface spray flux normalized by the water density ρ_w (m^3/m^2 s) is

$$\left.\frac{K}{\rho_w}\frac{d\mu}{dz}\right|_{z=0} = -\frac{\mu_0 w}{\rho_w \alpha_\mu}. \quad (9)$$

In the limit $w \longrightarrow 0$ (weightless admixture), $M \longrightarrow 0$, $h \longrightarrow \infty$, and we come to the solution for the admixture-free constant-flux layer [20]

$$du/dz = \lambda_T \alpha g Q/u_*^2, \quad d\theta/dz = \lambda_T \alpha g Q/\alpha_T u_*^4,$$

$$K = u_*^4/\lambda_T \alpha g Q, \quad b = u_*^2/s(\alpha_T\lambda_T)^{1/2},$$

$$l = (s^2\alpha_T/\lambda_T^3)^{1/4} u_*^3/\alpha g Q, \quad \mu = \mu_0.$$

Solution (8) is implicit so far because it is expressed through the as of yet unknown integration constants u_* and Q. We now assume that the values of horizontal velocity H and potential temperature u_H are specified at a certain height θ_H. The integration constants should be expressed through these values. From (8) we get an algebraic system for determining them:

$$u_H = \frac{\alpha_\mu u_*^2}{w}\left\{\frac{\lambda_T \alpha g w Q H}{\alpha_\mu u_*^4} + \ln\left[1 + M\left(1 - e^{-\frac{H}{h}}\right)\right]\right\}, \quad (10)$$

$$\Delta\theta_H = \theta_H - \theta_0 = \frac{\alpha_\mu Q}{\alpha_T w}$$
$$\times\left\{\frac{\lambda_T \alpha g w Q H}{\alpha_\mu u_*^4} + \ln\left[1 + M\left(1 - e^{-\frac{H}{h}}\right)\right]\right\}. \quad (11)$$

Dividing (10) by (11) yields

$$\frac{u_H}{\Delta\theta_H} = \frac{\alpha_T u_*^2}{Q}. \quad (12)$$

As is easy to verify, in a special heavy-admixture-free case $M = 0$, the values of u_H and $\Delta\theta_H$ are related by

$$u_H^2 = \alpha_T \lambda_T \alpha g \Delta\theta_H H \quad \text{или}$$
$$\mathrm{Ri} \equiv \frac{\alpha_T \lambda_T \alpha g \Delta\theta_H H}{u_H^2} = 1. \quad (13)$$

Here, the dimensionless parameter Ri has a sense of the integral "temperature" Richardson number in the layer discussed.

4. SOLUTION WITHOUT THERMAL EFFECTS

Contributions to density stratification come from the temperature and spray droplets (the ratio of these contributions at the surface is determined by the dimensionless parameter M). In hurricane winds, the temperature contribution cannot normally play a key role. This is evident, for example, from the estimate of the values of the thermal Richardson number given above in the introduction. For some problems, therefore, it makes sense to analyze a simplified version of the model in which density stratification is caused only by spray droplets. For this situation, there is a limiting transition $Q \longrightarrow 0$. In this limit, $h \longrightarrow \infty$, $M \longrightarrow \infty$; the solution takes the form

$$\mu = \frac{\mu_0}{1 + z/h_\mu}, \quad u = \alpha_\mu \frac{u_*^2}{w}\ln(1 + z/h_\mu), \quad (14)$$
$$K = \alpha_\mu^{-1} w h_\mu (1 + z/h_\mu),$$

$$l = (s^2\alpha_\mu\lambda_\mu)^{1/4}\frac{wh}{u_*}(1+z/h_\mu)$$
$$= \left(s^2\frac{\alpha_\mu}{\lambda_\mu^3}\right)^{1/4}\frac{\bar{\rho}u_*^3}{gw\mu_0}(1+z/h_\mu), \quad (15)$$

$$b = u_*^2/s(\alpha_\mu\lambda_\mu)^{1/2}, \quad N = \left(\frac{\lambda_\mu}{\alpha_\mu}\right)^{1/2}\frac{gw\mu_0}{\bar{\rho}u_*^2(1+z/h_\mu)}, \quad (16)$$

where the vertical scale $h_\mu = \frac{\alpha_\mu\bar{\rho}u_*^4}{\lambda_\mu gw^2\mu_0}$. is introduced. If the horizontal velocity u_H is prescribed at a certain height H, the integration constant u_* is found from the algebraic equation

$$u_*^2 = \frac{wu_H}{\alpha_\mu\ln(1+H/h_\mu)} = \frac{wu_H}{\alpha_\mu\ln\left(1+\frac{\lambda_\mu gw^2H}{\alpha_\mu u_*^4}S\right)}, \quad (17)$$

where $S \equiv \mu_0/\bar{\rho}$ is the surface mass concentration of spray droplets. Note that the Richardson number in this case is independent of both the height and the dimensional parameters of the problem: $\mathrm{Ri} = (\alpha_\mu\lambda_\mu)^{-1}$.

5. GENERALIZING THE MODEL TO THE CASE WHEN SPRAY MAKES A LARGE CONTRIBUTION TO THE DENSITY OF A MEDIUM

As in the previous works, it was assumed above that spray droplets make only a small contribution to the average air density: $\mu \ll \bar{\rho}$.. The spray-induced variations in air density were taken into account in the equations only where they were multiplied by the acceleration of gravity. As can be seen below, however, it would be reasonable to consider more general situations as well, when μ can be comparable to $\bar{\rho}$ or even exceed it significantly. As far as we know, there are rigorous models for such situations. It makes sense to consider the following generalization as a first step. In the system of equations used above, $\bar{\rho}$ is replaced by $\bar{\rho} + \mu$ (in (4) and (6)); therefore (if thermal effects are ignored), instead of $N = (-(g/\bar{\rho})(d\rho/dz))^{1/2}$, the following is considered:

$$N = \left(-\frac{g}{(\mu+\bar{\rho})}\frac{d\mu}{dz}\right)^{1/2}. \quad (18)$$

The balance equation of turbulent energy is written as

$$K\left(\frac{du}{dz}\right)^2 - \alpha_\mu KN^2 - K^3/c^4l^4 = 0, \quad (19)$$

where N is determined from (18). Eliminating all the unknowns except for b from the system of equations, we find $b = u_*^2/s(\alpha_\mu\lambda_\mu)^{1/2}$. With this taken into account, it is easy to obtain a nonlinear equation for the concentration of spray droplets

$$\frac{\mu+\bar{\rho}}{\mu^2}\frac{d\mu}{dz} = -\frac{\lambda_\mu gw^2}{\alpha_\mu u_*^4} = -\frac{1}{h_*}.$$

Here, the vertical scale is $h_* = \alpha_\mu u_*^4/(\lambda_\mu gw^2) = Sh_\mu$. The analytical solution is expressed implicitly

$$\ln\frac{\mu}{\mu_0} + \bar{\rho}\left(\frac{1}{\mu_0} - \frac{1}{\mu}\right) = -\frac{z}{h_*}, \quad \text{или}$$
$$\ln\frac{m}{m_0} + \left(\frac{1}{m_0} - \frac{1}{m}\right) = -Z, \quad (20)$$

where the dimensionless variables $m = \mu/\bar{\rho}$, $m_0 = \mu_0/\bar{\rho}$, $Z = z/h_*$ are introduced. Examples of the dimensionless vertical profiles of the concentration of spray droplets for different m_0 are given in Fig. 2.

At μ_0, $\mu \ll \bar{\rho}$, the logarithm on the right-hand side of (20) is insignificant, and we arrive at the solution of the previous section decreasing with height at scales on the order of h_μ.

At μ_0, $\mu \gg \bar{\rho}$ (possible only at sufficiently low heights), only the logarithm on the left-hand side of (20) is significant, and an approximate equation may be written explicitly:

$$\mu \approx \mu_0\exp\left(-\frac{z}{h_*}\right), \quad u \approx \lambda_\mu\frac{gw}{u_*^2}z,$$
$$K \approx \frac{1}{\lambda_\mu}\frac{u_*^4}{gw}, \quad l \approx \left(\frac{s^2\alpha_\mu}{\lambda_\mu^3}\right)^{1/4}\frac{u_*^3}{gw}, \quad (21)$$

$$N \approx \left(\frac{g}{h_*}\right)^{1/2} = \left(\frac{\lambda_\mu}{\alpha_\mu}\right)^{1/2}\frac{gw}{u_*^2}, \quad \mathrm{Ri} = (\alpha_\mu\lambda_\mu)^{-1}. \quad (22)$$

If the horizontal velocity H is specified at a certain height u_H, then

$$u_* = \left(\lambda_\mu\frac{gwH}{u_H}\right)^{1/2}, \quad h_* \approx \alpha_\mu\lambda_\mu\frac{gH^2}{u_H^2}, \quad u \approx u_H\frac{z}{H},$$
$$K \approx \lambda_\mu\frac{gwH^2}{u_H^2}, \quad l \approx (s^2\alpha_\mu\lambda_\mu^3)^{1/4}\left(\frac{gwH^3}{u_H^3}\right)^{1/2}, \quad (23)$$
$$N \approx (\alpha_\mu\lambda_\mu)^{-1/2}\frac{u_H}{H}.$$

The aerodynamic drag is

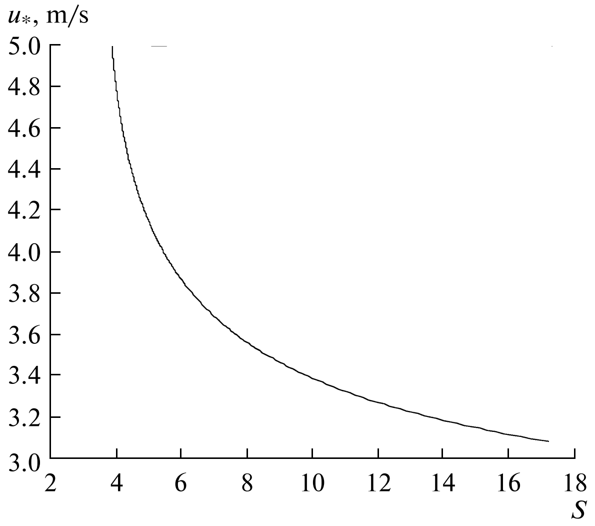

Fig. 3. Example of the dependence that the friction force has on the droplet concentration according to (25).

$$c_D \equiv \frac{u_*^2}{(u|_{z=10\text{ м}})^2} \approx \frac{\lambda_\mu g w H^3}{u_H^3 z_{10}^2} \quad (z_{10} = 10 \text{ м}). \tag{24}$$

6. ANALYSIS OF SOLUTIONS

When $M \ll 1$, the admixture (spray droplets) has no significant influence on the structure and dynamics of the near-surface layer. In accordance with this, solution (8) in the near-surface layer is close to the previous solution for the constant-flux layer free of admixture. In this case, $f(\xi) \approx 1$ and $\mu/\mu_0 \approx \exp(-\xi)$; perturbations are in the lower layer of thickness h. The spray penetration height h increases with a rise in the momentum flux (with u_*) and decreases with an enhancement of stable temperature stratification ($\alpha g Q$) and an increase in the particle sedimentation rate w.

As was already indicated, however, the asymptotics $Q \longrightarrow 0$, for which the solution has the form (14)–(17), are of greater interest in the given context. The concentration of spray droplets decreases with height at vertical scales on the order of h_μ. A stable vertical density gradient weakens turbulence at these scales. The near-surface values of K and l decrease as h_μ decreases (μ_0) increases), all other conditions being equal. This gives grounds to suggest the existence of positive feedback and the nonlinear self-closure of the heavy admixture near its source. The larger the vertical gradient of the surface concentration of spray droplets is, the more stable the density stratification is and the weaker the turbulence is. The latter circumstance, in turn, hampers the vertical admixture transfer, i.e., favors a higher concentration of admixture near its source.

From (17), we express the surface mass concentration of spray droplets

$$S \equiv \frac{\mu_0}{\bar{\rho}} \approx \frac{1}{\alpha_\mu \lambda_\mu g H} u_H^2 F^2[\exp(1/F) - 1] \approx \frac{u_H^2}{gH} F^2[\exp(1/F) - 1], \tag{25}$$

where the parameter $F = \alpha_\mu u_*^2/w u_H$ (dimensionless friction) is introduced; hereafter, the following values of the dimensionless constants are accepted for numerical estimates: $s = \alpha_T = \alpha_\mu = 1$ and $c^4 = 20$ [14–16, 22, 23]. Then, $\lambda_T = \lambda_\mu \approx 1$.

The right-hand side of (25) is a nonmonotonic function of its argument. With small values of the dimensionless friction F, it increases as F decreases. When $F \approx 0.6$, it has a minimum of about 1.5 and then increases. This means that, for the existence of a stationary solution, the water content in the model must be sufficiently large:

$$\frac{\mu_0}{\bar{\rho}} > 1.5\frac{u_H^2}{gH}. \tag{26}$$

This result is easy to interpret: when spray droplets have low concentrations, their stratification appears to be inadequate to compensate for the generation of turbulence at the large wind shears considered here. Inequality (26) is easy to relate to the effective integral Kolmogorov number [7, 17–19] or to analogues of the Froude and Richardson numbers.

Because of the nonmonotonicity of the right-hand side of (25), the friction–water content relationship $F(S)$ expressed by this equation is ambiguous. As can be easily verified, one of the two branches of solution (which fits large values of F) is inconsistent with some of the assumptions made in our model. We have assumed that the turbulence scale l has relatively small values, whereas the branch we are talking about fits sufficiently large values of u_* and, hence, l. Thus, we restrict our consideration to another branch of the solution, where $F < 0.6$. One example of the dependence that u_* has on $S = \mu_0/\bar{\rho}$ is given in Fig. 3 for $H = 100$ m, $u_H = 50$ m/s, and $w = 1$ m/s (which corresponds to a particle radius of about 100 μm.

As is seen from the figure, the friction force, in general, can decrease considerably as the concentration of spray droplets increases. In the model being considered, however, stationary solutions are possible only when spray-droplet concentrations are sufficiently large. Inequality (26) leads, in this case, to the limitation $\mu_0 > 4$ kg/m^3. This means, however, that we go beyond the commonly accepted assumption of a small contribution of spray droplets to air density. Therefore, it would be more logical to consider asymptotics (21)–(24), which is free of this assumption (though obtained at the cost of additional hypotheses). In this solution, the concentration of spray droplets falls exponentially with height; the wind speed grows linearly; and K, l, and N are height-independent.

The nontrivial properties of solution (23) and expression (24) are that the upper-boundary velocity u_H enters the denominators of the expressions for u_*, c_D, K, l, and h_* and the numerator of N, although inverse dependences are intuitively more appropriate. In other words, the friction in the model can indeed decrease as the velocity u_H increases.

Let $w = 1$ m/s and $u_H = 35$ m/s at $H = 10$ m. In this case, according to (23) and (24), we have $h_* \approx 0.8$ m, $u_* \approx 1.7$ m/s, $l \approx 0.5$ m, $K \approx 0.8$ m^2/s, $N \approx 3.5$ s^{-1}, and c^{-1}, $c_D \approx 0.0024$.

Thus, along with the dominant contribution that the heavy admixture (spray droplets) makes to the density of the medium, in accordance with the model, the significant weakening of turbulence and a decrease in the aerodynamic drag are indeed possible in principle. The last estimate, however, means that we are dealing with the surface-layer state, which is, in a sense, extreme and “exotic.” To suppress turbulence at the large wind shears considered here, extremely stable density stratification is required. In accordance with this, the buoyancy frequency in the last estimate was found to be two or three orders of magnitude greater than the values typical of the atmospheric boundary layer. According to (23), the corresponding period N^{-1} is on the order of H/u_H, the time during which the hurricane wind travels a very short distance, comparable to the near-water layer thickness H.

The turbulence scale l in such a stably stratified layer was found to be quite small. From physical considerations, as was already mentioned, it cannot exceed $\sqrt{b}/N$, which in the estimate indicated above accounts for only a few tens of centimeters. This contrasts sharply with the hypothesis $l \sim z$, which is commonly accepted in similar problems. True enough, hypothesis (5), which is accepted here and which assumes the smallness of l in comparison to the characteristic vertical scales of the near-water layer, is also most valid in some limit (only the condition $l < h_*$ is satisfied, while, in the strict sense, there must be $l \ll h_*$).

7. COMPARISON BETWEEN THE FIELD AND EXPERIMENTAL INTENSITIES OF SPRAY PRODUCTION

If the water content (the mass of spray droplets per unit volume) at the surface is μ_0 and the fall velocity of spray droplets is w, the absolute value of the downward mass flux of the spray is $w\mu_0$ (kg/m^2 s). In a stationary state, the intensity of spray production must be the same. If, for example, $\mu_0 = 2$ kg/m^3 and $w = 1$ m/s, the intensity must be 2 kg/m^2 s. The most recently published results of laboratory modeling [13] lead their authors to conclude that intensities of spray production are much lower (from Fig. 8 in [13], by two or three orders of magnitude less). The previous estimates also, as a rule, suggest similar conclusions. For example, various estimates of the spray flux given in Fig. 5 in [12] do not exceed the mass of a 70-mm water layer per hour, a value equivalent to about 2×10^{-2} kg/m^2 s. Thus, the spray fluxes are most likely insufficient for the proposed mechanism of turbulence suppression. At the same time, it should be noted that discrepancies between various data on such fluxes still remain very large. For example, summaries of the estimates in [7] (Fig. 6) and [11] (Figs. 1, 2), in general, do not rule out the possibility of a much higher intensity of spray production than the above-mentioned data. Therefore, the question probably cannot be considered completely closed. The results of this work give corresponding criteria that will be able to estimate the possible importance of the mechanism of turbulence suppression discussed here as the information about the intensity of spray production becomes more comprehensive.

8. CONCLUSIONS

In the framework of the suggested theoretical scheme, it is possible to find analytical solutions that can, in principle, describe nonlinear effects such as the self-closure of the heavy admixture near its source and the decrease in effective friction, which are related to turbulence suppression due to the strengthening of stratification stability. A significant decrease in turbulent friction in the model was made possible only at very high surface droplet concentrations (with the water content on the order of, or greater than, 1 l per cubic meter). Current data on the intensity of spray production, as a rule, do not support the possibility that such concentrations exist. In light of these results, alternative explanations of the decrease (saturation) in the coefficient of aerodynamic drag in hurricane winds seem to be more probable [8]. The results of this work give criteria for the efficiency of the proposed mechanism that can be used as the data on the intensity of spray production become more detailed.

This model uses a variety of hypotheses and assumptions. Apart from those mentioned above, we note some possible sources of errors. First, the semiempirical turbulence theory is particularly unreliable in a strong stable stratification, i.e., in situations that are of most interest in the context being considered. Second, this model ignores the additional weakening of turbulence caused by the inertia of heavy particles (droplets). The latter circumstance must enhance the effect under consideration, namely, a decrease in effective friction. There is, however, a factor that acts in the opposite direction which is not taken into account. The energy of the background wind is additionally spent on the horizontal acceleration of droplets that are sprayed into the surface layer. The surface-layer momentum sink related to this factor should be proportional to $\rho_d w \Delta u$, where ρ_d is the characteristic water content of the surface layer and Δu is the vertical difference of the velocity in it. If the latter product is compared with the explained part of the vertical momentum flux $\bar{\rho}u_*^2$, these quantities in the example considered here can, in general, be comparable.

Hypothesis (5) in these situations is probably a step forward in comparison with the commonly accepted hypotheses $l \sim z$, which strongly overestimate the turbulence scale when stratification is rather stable. The validity of (5), however, is also a debatable question. From physical considerations, it might be assumed that the role of wind shear at $z < l$ can be quite important, so there must be a lower sublayer with a logarithmic velocity profile. The closure scheme discussed here does not describe a limiting transition to the "logarithmic" layer. If data on the spray-production intensity sufficient for turbulence suppression ($\mu_0 \geq \bar{\rho}$) become available, the improvement of the model (an analysis of a two-layer model that considers adjacent layers with different properties and, hence, parameterizations) will make sense.

Despite using a number of hypotheses, the significance of the major conclusions of our work is supported by rather simple and reliable physical arguments. We consider the Froude number in the near-water layer

$$F = \frac{|\Delta\rho|}{\rho}\frac{gH}{u_H^2}.$$

Here, ρ and $\Delta\rho$ are the density of a medium and the vertical difference of this density. In order for the stable density stratification to considerably suppress turbulence, the Froude number must be about or greater than unity. Hence, we obtain a criterion

$$\frac{|\Delta\rho|}{\rho} \geq \frac{u_H^2}{gH}.$$

If $u_H = 30$ m/s at $H = 10$ m, the right-hand side of the last inequality is about ten. Thus, for turbulence suppression, very large vertical density differences, even slightly greater than those obtained in the calculations above, are in fact needed. Such differences can be ensured only if spray droplets make a dominant contribution to the surface-layer density. It follows that the intensity of spray production must be on the order of $w|\Delta\rho|$, and this is kilograms per square meter per second (the result that we have obtained in our work).

It should be noted that the near-surface layer model with the heavy admixture making the dominant contribution to the density of the medium might also be of interest in some other applications. In avalanches, for example, the density of the snow airflow in widespread situations is estimated at 5–10 kg/m^3 [24]. This value is close to some of the estimates considered above.

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