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On a Positive-Feedback Mechanism in Intense Atmospheric Vortices

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Abstract—Attention is focused on a positive feedback that may play a significant role in intense vortices, such as tornadoes and, probably, tropical cyclones: rotation suppresses turbulence which, in turn, may intensify rotation. Some simple models illustrate this phenomenon.

Keywords: tornado, tropical cyclones, turbulence, rotation, laminarization, positive feedback
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1. INTRODUCTION

Intense atmospheric vortices (tornadoes and tropical cyclones) are characterized by a cyclostrophic balance: an approximate equilibrium between the pressure-gradient force directed to the axis of rotation and the sum of both centrifugal and Coriolis forces (the latter is usually negligibly small in the central regions of intense vortices). This balance is disturbed, first and foremost, by dissipative factors (viscosity and heat conductivity). If velocity shears in such vortices are significant, turbulence and its related vortex dissipation are, at first glance, bound to be very intense and efficient. For example, if the horizontal size of a whirlwind is $D = 30$ m and the coefficient of turbulent exchange is $K = 10$ m^2/s, the conventional estimation of the characteristic time of diffusion processes D^2/K yields the time of vortex dissipation on the order of 1 min. However, such vortices usually live noticeably longer and, even if they do not live long, the pattern observed usually differs from classical diffusion: a sharply pronounced vertical (or inclined) vortex boundary is observed [1, 2]. Such gradients (jumps) seem to be incompatible with intense turbulent mixing.

As an explanation, one can assume that some features of the interaction between rotation and turbulence are important for intense vortices. It is known that intensive rotation, as well as stable stratification, can noticeably suppress turbulence [3–7]. Therefore, the following mechanism of positive feedback is noticeable. Rapid rotation suppresses turbulence; this, in turn, favors the maintenance and intensification of the cyclostrophic regime with a high-speed rotation and horizontal pressure and buoyancy differences. Below, this is illustrated on the basis of some simple models.

2. ESTIMATING THE CHARACTERISTIC TIME OF TURBULENCE SUPPRESSION IN A WHIRLWIND

Because of the known analogy between the effects of rotation and stratification, in order to describe the effect of rapid rotation on turbulence, it is reasonable to use models tested in describing stratification effects. In particular, in this work, a corresponding semi-empirical turbulence model [8], which is in good agreement with the similarity theory under sufficiently stable stratification [9], is modified in order to describe the effects of rapid rotation.

A priori one can expect that the turbulence scale l decreases with increasing stratification (rotation). Let us consider the extreme case, when l is much smaller than characteristic horizontal scales on which the mean specific kinetic energy of turbulent fluctuations b and other mean (nonturbulent) fields noticeably change. In this case, in the Kolmogorov–Monin equation of turbulent-energy balance [10], the diffusion summand is negligibly small when compared to the dissipative one (this can be verified, for example, when the solution for b is found). In this case, the abovementioned equation can be written in the form

$$\frac{db}{dt} = -\alpha_T K N^2 - \frac{K^3}{c^4 l^4} + B, \quad K = l\sqrt{b}. \tag{1}$$

Here, t is time, N is the buoyancy frequency, K is the coefficient of turbulence, c and α_T are the dimensionless parameters (we consider them constant in the simplest model), and the summand B describes turbulent-energy generation.

As for the turbulence scale l, let us advance the following hypothesis [8]:

$$l = sb^{1/2}/N, \tag{2}$$

where s is the dimensionless constant. Equation (2), often called the Ozmidov scale, has a simple physical meaning: the right-hand side of (2) with an accuracy of the constant multiplier is the distance vertically traveled by a medium particle with the initial turbulent velocity $b^{1/2}$, before this particle is brought to rest by buoyancy forces. It is clear that the turbulence scale cannot exceed the abovementioned distance in order of magnitude. When this distance is short (under sufficiently stable stratification), there is no reason for taking l smaller than the right-hand side of (2). This hypothesis is in agreement also with both dimensionality and similarity considerations [9].

In a rotating medium, the so-called parameter of inertial stability is an analog of buoyancy frequency (see, for example, [11] and references there):

$$N_v = \sqrt{\left(\frac{2v}{r} + f\right)\left(\frac{v}{r} + \frac{\partial v}{\partial r} + f\right)}. \quad (3)$$

Here, r is the distance to the axis of vortex (which is assumed to be axisymmetric), v is the tangential velocity, and f is the Coriolis parameter. In the simplest case of solid-body rotation (a situation characteristic of the central regions of geophysical vortices),

$$N_v = \frac{2v}{r} + f \approx \frac{2v}{r} \quad (4)$$

and it does not depend on radial coordinate (it is determined by angular velocity).

If, in order to describe the dynamics of turbulence in the core of vortex, we use system (1), (2), replacing N by N_v [12], it will be easy to estimate the characteristic time of suppressing turbulence due to intensive rotation. The abovementioned system is reduced to the equation

$$\frac{d(N_v K)}{dt} + SN_v^2 K = sB, \quad (5)$$

in which $S = s\alpha_T\left[1 + 1/\left(c^4 s^2 \alpha_T\right)\right]$ is the dimensionless constant. In this case,

$$b = \frac{1}{s} N_v K, \quad l = (sK/N_v)^{1/2}.$$

Let the turbulence field at the initial time be specified as $K|_{t=0} = K_0$. Then, if the turbulence generation (which is absent under solid-body rotation) is neglected, it is easily seen from (5) that turbulence will be suppressed over the time on the order of $(SN_v)^{-1} \approx S^{-1}(r/2v) = (2S\omega)^{-1}$, where ω is the angular velocity of rotation. One can apparently assume that the dimensionless parameter S is on the order of unity; then the characteristic time of turbulence suppression is comparable to the period of vortex-core rotation or it is one order of magnitude longer. For example, this characteristic time may amount to a few seconds for tornadoes and a few tens of minutes for tropical cyclones. These estimates are rough; however, they clearly demonstrate the efficiency of the mechanism of suppressing turbulence in the cores of intense vortices.

3. TURBULENCE DIFFUSION FROM THE HIGHEST WIND REGION TO THE "EYE" OF A TROPICAL CYCLONE

In the vicinity of the radius of the highest winds, the gradient $\partial v/\partial r$ rapidly changes, which implies variations in N_v, i.e., in the conditions for the occurrence of turbulence. For example, in tropical cyclones (TCs), the highest wind region is usually not so far (tens of kilometers) from the axis of rotation. In the vicinity of this region, within which the rotation is already not solid-state and velocity shears are significant, turbulence is, no doubt, intensively generated. This may raise the question of how far this turbulence can penetrate in the direction of the axis of rotation. The simplest (but conceptual) model should describe the nonlinear diffusion of turbulent energy into the central vortex region, within which there are sinks of turbulent energy that are associated with its dissipation and with turbulence suppression due to intensive rotation.

Let us assume that, at the outer boundary of the central vortex region under consideration—a circle with the radius r_m, within which the wind velocity is close to its maximum v_m—the specific kinetic energy of turbulent fluctuations is specified as b_m. Inside this region, instead of (1), we consider the steady-state equation with additional consideration for nonlinear diffusion:

$$-\alpha_T K N_v^2 - \frac{K^3}{c^4 l^4} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\alpha_b K \frac{\partial b}{\partial r}\right) = 0, \quad (6)$$

where α_b is the dimensionless parameter.

In the left-hand side of (6), let us write down the latter summand in the form of the sum:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\alpha_b K \frac{\partial b}{\partial r}\right) = \frac{\partial}{\partial r}\left(\alpha_b K \frac{\partial b}{\partial r}\right) + \frac{1}{r}\left(\alpha_b K \frac{\partial b}{\partial r}\right). \quad (7)$$

Let the radial scale—the characteristic depth of turbulence penetration from the outer boundary of the region to the axis of rotation—be denoted by Δr. Let us consider the case $\Delta r \ll r_m$ (it will be shown below that this case is of special interest). In this case, a scale analysis gives grounds to neglect the latter summand in the right-hand side of (7). In fact, when compared to the former summand, this latter is of the order $\Delta r/r_m \ll 1$. In this approximation, Eq. (6) takes on the form

$$-\alpha_T K N_v^2 - \frac{K^3}{c^4 l^4} + \frac{\partial}{\partial r}\left(\alpha_b K \frac{\partial b}{\partial r}\right) = 0. \quad (8)$$

The meaning of this simplification is quite clear. If turbulence penetrates deep into only a thin ring

$$r_m - \Delta r \le r \le r_m, \quad (9)$$

the effects of polar geometry are nonprincipal (radial coordinate within this ring slightly varies) and the diffusion summand slightly differs from that for the case of plane geometry, for which a number of problems were studied in [13]. With consideration for (2) and the hypothesis $K = l\sqrt{b}$, Eq. (8) can easily be reduced to the form

$$\frac{d^2\left(b^2\right)}{dr^2} - S_1^2 N_v^2 b = 0, \quad (10)$$

where the dimensionless parameter is

$$S_1 = \left[\frac{2\alpha_T}{\alpha_b}\left(1 + \frac{1}{c^4 s^2 \alpha_T}\right)\right]^{1/2}.$$

In the region of solid-body rotation, where $N_v = \text{const}$, (10) is the nonlinear equation with constant coefficients, which admits a general analytical solution. First and foremost, note that there is an identically zero solution of this equation. A special feature of such problems of the non-linear-diffusion (heat-conductivity) type is in the presence of a sufficiently clear boundary of the disturbed region: a nonzero solution that decreases with distance from the source and vanishes at a finite distance, where this solution is joined to the identically zero solution (see, for example, [13] and references there).

Let us introduce the variables $\psi \equiv b^2$, $\eta \equiv d\psi/dr$. Then

$$\frac{d^2\left(b^2\right)}{dr^2} = \frac{d\eta}{dr} = \frac{d\eta}{d\psi}\frac{d\psi}{dr} = \eta\frac{d\eta}{d\psi} = \frac{1}{2}\frac{d\left(\eta^2\right)}{d\psi}.$$

Equation (10) takes on the form

$$\frac{d\left(\eta^2\right)}{d\psi} - 2S_1^2 N_v^2 \sqrt{\psi} = 0.$$

Integrating, we obtain

$$\eta^2 = \frac{4}{3} S_1^2 N_v^2 \psi^{3/2} + C,$$

where C is the integration constant that should be assumed to be zero, because we suppose (this will be verified below) that, with distance from the outer boundary, both ψ and η (i.e., turbulent energy and its flux) are bound to vanish. Then

$$\eta \equiv \frac{d\psi}{dr} \equiv \frac{d\left(b^2\right)}{dr} = \pm\frac{2S_1 N_v}{\sqrt{3}} b^{3/2}.$$

At $b \ne 0$, hence, it follows that

$$\frac{db}{dr} = \frac{S_1 N_v}{\sqrt{3}} b^{1/2}$$

(sign is chosen with consideration for a decrease in the energy of turbulence with a decrease in r). Integrating with consideration for the edge condition at the outer boundary $r = r_m$ after the abovementioned joining with the zero solution yields

$$b = \begin{cases} b_m\left[1 - \dfrac{S_1 N_v}{2\sqrt{3b_m}}(r_m - r)\right]^2, & \text{при } r_m - \Delta r \le r \le r_m, \\ 0, \text{при } 0 \le r \le r_m - \Delta r. \end{cases} \quad (11)$$

Here, the following designation is introduced:

$$\Delta r = \frac{2\sqrt{3b_m}}{S_1 N_v}. \quad (12)$$

The meaning of Δr is similar to that of the abovementioned depth of turbulence penetration from the outer boundary of the region to the axis of rotation. In accordance with the foregoing, $N_v \approx 2v_m/r_m$, then it follows from (12) that

$$\frac{\Delta r}{r_m} = \frac{\sqrt{3b_m}}{S_1 v_m}.$$

In order of magnitude, $\sqrt{b_m}$ coincides with the characteristic scale of the velocity of turbulent fluctuations v_t on the radius r_m. If we assume that the parameter S_1 is of the order of unity, then

$$\frac{\Delta r}{r_m} \sim \frac{v_t}{v_m}.$$

The latter ratio (between the velocity of turbulent fluctuations and the maximum wind velocity in a tropical cyclone) is much less than unity. This implies the validity of the above-assumed inequality $\Delta r \ll r_m$. In other words, in the model under consideration, rotation suppresses turbulence so efficiently that intense turbulence in the region of maximum winds penetrates in the direction of the axis of rotation only to a very small degree. This circumstance can, to some extent, explain the nontrivial fact of the absence of strong windflaws in the eye of TC. In the available numerical models of TC, turbulence seems to be too roughly described for characterizing similar effects.

4. A MODEL THAT ILLUSTRATES THE INFLUENCE OF TURBULENCE ON ROTATION

As a simple illustration of the inverse effect of turbulence on rotation, let us consider the linear stationary problem of generating vortex motions by a volume buoyancy source in a viscous rotating stratified medium. (In literature, there is an opinion that the role of the volume sources of heat (buoyancy) in whirlwinds is significant [1]; the role of heat generation due to phase transitions in tropical cyclones is commonly known.)

Let a specified stationary volume source of buoyancy (heat) operate in an unbounded uniformly rotating stratified medium. For simplicity, let us limit ourselves to a two-dimensional stationary problem, in which the source Q (K/s) and disturbances produced by this source depend on one of the horizontal coordinates x and the vertical coordinate z. The linearized system of equations in the Boussinesq approximation has the form

$$0 = -\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) + f v, \tag{13}$$

$$0 = \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right) - fu, \quad \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{14}$$

$$0 = -\frac{\partial P}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) + \alpha g \theta, \tag{15}$$

$$w\gamma = \kappa\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial z^2}\right) + Q. \tag{16}$$

Here, u, v, w are the components of wind velocity along the horizontal axes x, y, and the vertical axis z, respectively; P is the pressure disturbance normalized to the density of medium; $f = 2\Omega$ is the Coriolis parameter; Ω is the angular velocity of the "background" rotation of medium; θ is the disturbance of temperature (potential temperature in the air); ν and κ are the exchange coefficients; α is the coefficient of thermal medium expansion; and γ is the background vertical gradient of potential temperature. The background stratification may be both stable ($\gamma > 0$) and unstable ($\gamma < 0$). (However, in the latter case, one should consider a medium layer vertically bounded in order to provide the convective stability of the background state.)

For the purposes of illustration, it will suffice to consider the simplest case of a buoyancy source harmonically dependent on coordinates $Q = Q_0 \cos(k_x x)\cos(k_z z)$, where the constant Q_0 is the amplitude and k_x, k_z are the corresponding wavenumbers. We also seek solutions for disturbances in the harmonic form. The system of equations is reduced to the linear algebraic system for amplitudes; here is the result:

$$u = \frac{k_z}{k_x}\frac{\alpha g Q_0}{\kappa\nu k^4} F \sin(k_x x)\sin(k_z z),$$

$$v = -\frac{k_z}{k_x}\frac{\alpha g Q_0}{\kappa\nu k^4} F\sqrt{\mathrm{Ta}} \sin(k_x x)\sin(k_z z),$$

$$w = \frac{\alpha g Q_0}{\kappa\nu k^4} F \cos(k_x x)\cos(k_z z),$$

$$\theta = \frac{Q_0}{\kappa k^2}\frac{1 + (k_z/k_x)^2(1+\mathrm{Ta})}{1 - \mathrm{Ra} + (k_z/k_x)^2(1+\mathrm{Ta})} \times \cos(k_x x)\cos(k_z z),$$

$$P = \frac{k_z}{k_x^2}\frac{\alpha g Q_0}{\kappa k^2}\frac{1+\mathrm{Ta}}{1 - \mathrm{Ra} + (k_z/k_x)^2(1+\mathrm{Ta})} \times \cos(k_x x)\sin(k_z z),$$

where $F = \left[1 - \mathrm{Ra} + (k_z/k_x)^2(1+\mathrm{Ta})\right]^{-1}$,

$\mathrm{Ra} = -\alpha g\gamma/\kappa\nu k^4$, $\quad \mathrm{Ta} = \left(2\Omega/\nu k^2\right)^2$, $\quad k^2 = k_x^2 + k_z^2$.
This consideration is limited by parameter values at which the condition of the convective stability of background state $1 - \mathrm{Ra} + (k_z/k_x)^2(1 + \mathrm{Ta}) > 0$; is fulfilled; as the stability boundary is approached, the amplitudes of linear disturbances unrestrictedly increase.

The effect of the source of buoyancy results in the occurrence of convection; convective motions occurring in a rotating medium begin to spin. It is interesting to note the nonmonotonic dependence of the velocity of occurring vortex motion v on the Taylor number Ta. As Ta increases, the velocity v initially increases; however, at

$$\mathrm{Ta} = 1 + (1 - \mathrm{Ra})\left(\frac{k_x}{k_z}\right)^2$$

it reaches maximum and then decreases.

The ratio between occurring and background vorticities at large Taylor numbers is expressed by the relation

$$\left|\frac{(\partial v/\partial x)}{\Omega}\right| \sim \frac{2k_z\alpha g Q_0}{\kappa k^2}\frac{1}{\left[2(k_z/k_x)\Omega\right]^2 + (\nu/\kappa)N^2},$$

where $N^2 = \alpha g\gamma$. It is seen that, under turbulent exchange ($\nu \approx \kappa$), even under stable stratification ($N^2 > 0$), with a decrease in the coefficients of exchange, the occurring vorticity (in the linear model under consideration) unrestrictedly increases. If stratification is insignificant, the last ratio is simplified:

$$\left|\frac{(\partial v/\partial x)}{\Omega}\right| \sim \frac{\alpha g Q_0 k_x^2}{2\kappa k^2\Omega^2 k_z}. \tag{17}$$

Let, for example, $k_x = 0.1\ \mathrm{m}^{-1}$, $k_z = 0.01\ \mathrm{m}^{-1}$ (let characteristic horizontal and vertical source sizes be a few tens of meters and a few hundreds of meters, respectively), $\Omega = 0.3\ \mathrm{s}^{-1}$ (background-rotation period on the order of 20 s), $Q_0 \sim 1$ K/min (very moderate source intensity, if the fact that the lifetime of a whirlwind most often amounts to only a few minutes is taken into account), and $\kappa = 1\ \mathrm{m}^2/\mathrm{s}$. In this case (17) approaches unity in order of magnitude. Further suppression of turbulence even more strongly intensifies rotation, but this is beyond the validity range of the linear model under consideration.

It follows from the given estimate that even a very moderate attenuation of turbulence can noticeably intensify rotation, which, in turn, can (as was mentioned above) result in further turbulence suppression. It is possible that the role of this positive feedback in the dynamics of intense atmospheric vortices is significant.

It also follows from (17) that vertically extended vortices (to which small values of k_z and large values of k_x correspond) intensify most effectively. This is in agreement with the observed form of whirlwinds, dust devils, etc. In recent years the occurrence of vertically extended intense convective vortices in the wall of TC's eye (misocyclones [14]) has been observed. In the light of the above-given solution, the generation of extremely intense vortices with small horizontal scales under convection against the background of a vortex with a larger scale seems natural.

The extension of applicability of the hydrostatic approximation under rapid-rotation conditions, which also follows from the obtained solution, seems nontrivial. In fact, it is usually considered that this approximation is applicable for cases when the horizontal scales of such a process are much larger than its vertical scales, i.e., at $k_z/k_x \gg 1$. In the solution found, if $\mathrm{Ta} \gg 1$,, then

$$\frac{\partial P/\partial z}{\alpha g \theta} = \frac{(k_z/k_x)^2 (1 + \mathrm{Ta})}{1 - \mathrm{Ra} + (k_z/k_x)^2 (1 + \mathrm{Ta})}$$

is close to unity at significantly softer limits on spatial scales. For example, under neutral stratification, the condition of applicability of hydrostatics has the form $k_z/k_x \gg 1/\sqrt{\mathrm{Ta}}$; a vertically extended vortex may, nevertheless, be hydrostatic!

5. CONCLUSIONS

The above analytical models illustrate the probable efficiency of the positive feedback under consideration. The results of analysis of the semi-empirical equation of turbulent-energy balance show that, in the core of an intense vortex, turbulence is suppressed over time comparably to the characteristic period of rotation, i.e., very effectively. This circumstance hampers the penetration of turbulence "from the outside" in the direction of the axis of rotation, which, in particular, clearly explains the observed absence of strong windflaws in the eye of a TC. The suppression of turbulence, in turn, intensifies rotation.

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