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Toward a Nonlinear Theory of Katabatic Winds

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Abstract—The classic Prandtl slope flow model is generalized to include nonlinear turbulent friction and rotation. Several general regularities are established. In particular, a universal expression for the mass flux along the slope and a relationship between the surface velocity components, both independent of the friction law, are obtained. The applicability of the model to describing katabatic winds on fairly large horizontal scales is discussed.

Keywords: slope flows, Prandtl model, nonlinear flow, rotation, analytical solutions.

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An extensive literature is devoted to the theory of flows above a cooled or heated inclined surface [1–11]. One of most important application fields of this theory is katabatic winds in the atmosphere. The recent investigations have mainly been focused on the numerical simulation of specific situations. Nevertheless, many fairly general questions still remain unanswered and not all important physical mechanisms are studied even in a maximally simple form. Moreover, very few analytical solutions are known, although such solutions could be checked and analyzed directly, make it possible to establish general regularities, and play the role of test examples for numerical calculations. This is especially true for nonlinear problems.

In [10], the classic steady-state Prandtl slope flow model was generalized to include the case of quadratic turbulent friction and heat transfer. It was shown that this generalization leads to qualitatively new properties of the solutions. It is of interest to consider an analogous problem with account for the Coriolis accelerations, which substantially advances the model to describing large horizontal scales, such as scales of Antarctic katabatic winds.

It is known that nowadays there are no analytical models of slope flows with account for all the above-mentioned factors. As a certain analog, we should mention paper [9] which contains a "zero-dimensional" model neglecting the attitude dependence of the wind velocity. This models seems to be a necessary step but there is a need in models in which the viscous effects would be described in the traditional hydrodynamic form. The rotation effects were considered in [11], but only for a problem with the no-slip boundary conditions. The present study is aimed at filling up the gap: we will investigate a model in which both the rotation effects and nonlinear friction are taken into account. The one-dimensional steady-state problem, in which the dependence on the z coordinate normal to the slope is only important, will be considered.

1. FORMULATION OF THE PROBLEM

The problem geometry is schematically shown in Fig. 1. A semibounded medium (air, to be specific) is cooled from below and the cooled layer "flows down" the slope under the action of its own gravity. The Coriolis accelerations lead to a deviation of the flow direction from the y axis directed downward along the slope. To be specific, we will consider the Southern hemisphere where the Coriolis parameter $f < 0$ and the Coriolis forces are directed leftward with respect to the motion direction. The stationary one-dimensional

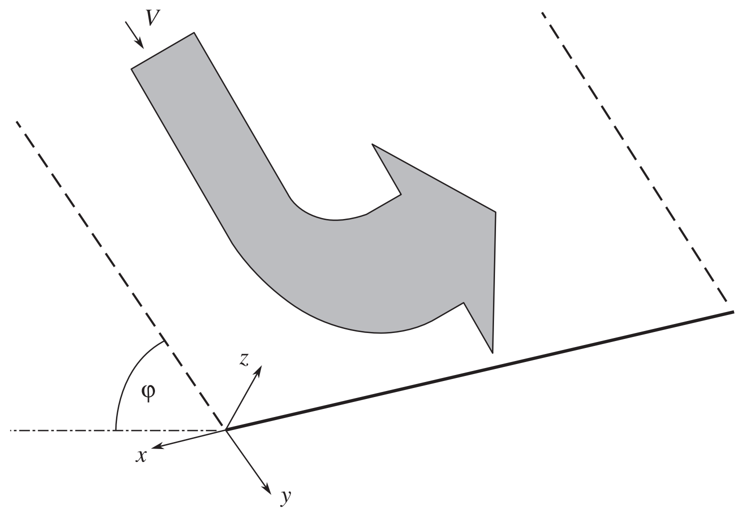

Fig. 1. Diagram of the problem geometry.

equations describing the slope flow can be represented in the form [1, 5, 9]:

$$0 = \nu \frac{d^2u}{dz^2} + fv\cos\varphi,$$

$$0 = \nu \frac{d^2v}{dz^2} - fu\cos\varphi - \alpha g\theta\sin\varphi, \quad (1.1)$$

$$-\gamma v\sin\varphi = \kappa\frac{d^2\theta}{dz^2}.$$

Here, the z axis is perpendicular to the subsurface; u and v are the velocity components along the x and y axes whose orientation on the sloping subsurface is shown in Fig. 1; θ is the temperature deviation from a background, the potential temperature for air [12]; γ is the vertical background gradient of the potential temperature; ν and κ are transfer coefficients; φ is the angle of inclination of the subsurface with respect to the horizontal plane; $\alpha = 1/T_0$ is the thermal expansion coefficient of the medium; T_0 is the average reference air temperature; and g is the gravity acceleration. We note that in [9] the analogs of the first two equations (1.1) did not contain the multiplier $\cos\varphi$. This seems to be a mistake but in many cases, for example, for katabatic winds on large horizontal scales, the angles φ are relatively small and $\cos\varphi$ differs from unity only slightly.

On the lower boundary $z = 0$ we will assume for the velocity components the quadratic friction conditions which are fairly often used in contemporary atmospheric models for describing turbulent friction [10, 12–14]

$$\nu\frac{du}{dz} = c_D u\sqrt{u^2 + v^2}, \quad \nu\frac{dv}{dz} = c_D v\sqrt{u^2 + v^2} \quad (z = 0), \quad (1.2)$$

where c_D is a dimensionless resistance coefficient. The fact that we waive the no-slip boundary conditions means that the boundary conditions are assigned not on the rigid surface but at a certain distance from it. This distance cannot always be determined unambiguously. In atmospheric models, in the case of a horizontal surface, this attitude is usually of the order of 10 m. It is important that in many cases the results depend on this distance only slightly. Thus, we can leave this question open. On the inclined boundary we specify a constant perturbation of the heat flux Q

$$c_p\rho\kappa\frac{d\theta}{dz} = Q \quad (z = 0), \quad (1.3)$$

where c_p is the heat capacity and ρ is the air density. To be specific, we will restrict consideration to the case of cooling from below, which corresponds to $Q > 0$. In principle, the case of heating from below with $Q < 0$ and motion up the slope can also be considered. The results will coincide correct to the sign. However, this symmetry takes place only for not too high $|Q|$. If the heating from below is sufficiently intense, the convective stability of the medium may be violated, which in its turn means that the model considered is inapplicable. In [1, 11], on the lower boundary the deviation of the temperature, not the heat flux, was specified. Both variants are limiting cases and it would be useful to investigate them.

Far from the inclined surface we assign the conditions

$$v \to 0, \quad \frac{du}{dz} \to 0, \quad \frac{d\theta}{dz} \to 0 \quad (z \to \infty). \tag{1.4}$$

2. SOLUTION

Preliminarily, we will note that the profiles $u(z)$ and $\theta(z)$ are similar, which follows from the first and third equations of system (1.1) and boundary conditions (1.4):

$$\frac{du}{dz} = \frac{\kappa}{\nu}\frac{f}{\gamma}\cot\varphi\,\frac{d\theta}{dz}. \tag{2.1}$$

From this, in view of (1.3), there follows the relation

$$\frac{du}{dz} = \frac{Qf}{c_p\nu\rho\gamma}\cot\varphi \quad (z = 0). \tag{2.2}$$

Equality (2.1) is also valid in a more general case of attitude-dependent transfer coefficients if the perturbations of the corresponding diffusion fluxes tend to zero with attitude.

Eliminating from system (1.1) all unknowns, except for the velocity component parallel to the slope v, we arrive at the equation

$$\frac{d^4v}{dz^4} + \frac{4}{h^4}v = 0, \tag{2.3}$$

where the attitude scale

$$h = \frac{\sqrt{2}}{\sqrt[4]{(\alpha g\gamma/\kappa\nu)\sin^2\varphi + (f^2/\nu^2)\cos^2\varphi}} = \frac{\sqrt{2}\sqrt[4]{\nu\kappa}}{\sqrt{N\sin\varphi}}\frac{1}{\sqrt[4]{1+\varepsilon}}, \quad \varepsilon = \frac{\kappa}{\nu}\frac{f^2}{N^2}\cot^2\varphi. \tag{2.4}$$

Here, $N = (\alpha g\gamma)^{1/2}$ is the Brent–Wyaisyal buoyancy frequency. We note that, correct to the multiplier $(\kappa/\nu)\cos^2\varphi$ close to unity, the dimensionless parameter ε is inverse to the parameter E used in [9]. In Antarctica E varies from 0.1 in the heart of the continent to 10 in the littoral regions [9]. Since the initial equations of this study coincide with those in [11], formulas (2.3) and (2.4) coincide, correct to notation, with the corresponding formulas in [11] where the value equivalent to ε is always assumed to be a small parameter. In addition to other generalizations, the present study is free from this restriction.

Taking into account that v is damped at large z, the solution of (2.3) can be written in the form

$$v = \left(C_1\sin\frac{z}{h} + C_2\cos\frac{z}{h}\right)\exp\left(-\frac{z}{h}\right),$$

where C_1 and C_2 are integration constants and $C_2 = v_0$. Here, the subscript "0" denotes values on the lower boundary. The velocity values are normalized to the scale $W = 2(Q/Q_b)\kappa/h\sin\varphi$, where $Q_b = c_p\rho\kappa\gamma$ is a vertical "background" heat flux. In the dimensionless variables $U = u/W$, $V = v/W$, and $T = \theta/(\gamma hQ/Q_b)$

the solution can be represented in the form:

$$U = -\sqrt{\frac{\varepsilon}{1+\varepsilon}}\left\{1 - V_0 + \left[-\left(1 - V_0\right)\cos\frac{z}{h} + V_0 \sin\frac{z}{h}\right]\exp\left(-\frac{z}{h}\right)\right\} + U_0,$$

$$V = \left[\left(1 - V_0\right)\sin\frac{z}{h} + V_0 \cos\frac{z}{h}\right]\exp\left(-\frac{z}{h}\right), \tag{2.5}$$

$$T = \left[-\left(1 - V_0\right)\cos\frac{z}{h} + V_0 \sin\frac{z}{h}\right]\exp\left(-\frac{z}{h}\right) - \varepsilon\left(1 - V_0\right) + \sqrt{\varepsilon(1+\varepsilon)}\,U_0.$$

For the mass flux along the slope we obtain the universal expression

$$\Pi = \int_0^\infty v\,dz = \frac{1}{2}hW = \frac{Q}{c_p\rho\gamma\sin\varphi} = \frac{Q}{Q_b}\frac{\kappa}{\sin\varphi}, \tag{2.6}$$

which is independent of the friction law and the presence of rotation. As can be easily verified, this expression also holds in the classical Prandtl solution with the no-slip condition in the absence of rotation [1]. From (2.6) it follows that the expression for the slope-ward flow velocity average over attitude is also universal:

$$\langle v\rangle = \frac{\Pi}{h} = \frac{W}{2} = \frac{Q}{c_p\rho\gamma h\sin\varphi} = \frac{Q\kappa}{Q_b h\sin\varphi}. \tag{2.7}$$

In order to find the surface values of the dimensionless velocity components U_0 and V_0, we will use boundary conditions (1.2). We arrive at the algebraic system

$$-A = U_0\sqrt{U_0^2 + V_0^2}, \tag{2.8}$$

$$B(1 - 2V_0) = V_0\sqrt{U_0^2 + V_0^2}, \tag{2.9}$$

where the dimensionless parameters

$$A = \frac{\nu}{\kappa}\frac{Q_b}{Q}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\frac{\sin\varphi}{2c_D}, \quad B = \frac{\nu}{\kappa}\frac{Q_b}{Q}\frac{\sin\varphi}{2c_D}, \quad \frac{A}{B} = \sqrt{\frac{\varepsilon}{1+\varepsilon}}. \tag{2.10}$$

In principle, we can reduce system (2.8), (2.9) to an algebraic fourth-order equation and write down the solution in a closed, though very cumbersome, form. However, some important features of the solution can be seen from a simpler analysis.

3. ANALYSIS OF THE SOLUTION

We should comment the fact that, as can be seen from (2.6) and (2.7), the mass flux and the established slope-ward flow velocity increase, do not decrease, with decrease in the slope steepness (angle of inclination φ). In the theory of slope flows other regularities are known, paradoxical at first sight, whose nature is discussed in, for example, [1, 8, 9, 15].

In the classical Prandtl problem [1, 15], for the temperature deviation given on $z = 0$, the established maximum and average velocities do not depend on the slope steepness. At first sight, this seems to be a mistake because of the absence alone of a passing to the rest state limit at $\varphi = 0$. The thing is that as $\varphi \to 0$ the effective thickness of the downflowing medium layer tends to infinity in the stationary problem without rotation and also strongly increases in the problem with rotation. Hence, with decrease in φ not only the driving force decreases but also the retarding viscous force, so that in the Prandtl problem the maximum velocity does not change. Moreover, with increase in h the stationary solution establishment time increases and at $\varphi = 0$ there is no stationary solution. Therefore, there is also no transition to a limiting rest state as $\varphi \to 0$ and $h \to \infty$. For sufficiently small angles φ it is reasonable to consider nonstationary problems.

In the present problem, on the lower boundary the steady-state heat flux perturbation is given, not the temperature perturbation $\Delta\theta$ as in the classical Prandtl problem. Therefore, with decrease in φ and increase in h the surface temperature deviation also increases:

$$\Delta\theta \sim h\frac{\partial\theta}{\partial z} \quad (z = 0).$$

Therefore, as follows from the solution, with decrease in the slope steepness not only the mass flux increases but also the maximum and attitude-average velocities.

As in [11], in contrast to the classical Prandtl problem, the steady-state temperature perturbation and the velocity component perpendicular to the slope u are not, in general, damped with attitude but tend to constant values to be analyzed below.

In the Southern hemisphere, $f < 0$ and $A > 0$ and from (2.8) it follows that $U_0 < 0$. From (2.8) and (2.9), certain general restrictions on the amplitudes of the velocity components can be seen. From (2.9) and (2.10) it follows that the signs of V_0 and $1 - 2V_0$ are the same. From a physical consideration, $V_0 > 0$ (Fig. 1) and, hence,

$$0 \le \mathrm{v}_0 \le W/2. \tag{3.1}$$

From (2.8) there follows a restriction on the other velocity component

$$\left|U_0\right| \le \sqrt{A} \quad \text{or} \quad \left|u_0\right| \le \sqrt{\frac{|f|h\cos\varphi}{2c_D}}. \tag{3.2}$$

From the same equation a restriction from below can also be easily obtained: the absolute value of the velocity components U_0 and V_0 cannot simultaneously be of smaller order than $\sqrt{A}$. Hence, in any case, the order of the surface wind velocity module is not smaller than $\sqrt{A}W$.

Dividing (2.9) by (2.8), we obtain

$$\frac{V_0}{U_0} = -\sqrt{1 + \frac{1}{\varepsilon}}\left(1 - 2V_0\right) \quad \text{or} \quad U_0 = -\sqrt{\frac{\varepsilon}{1 + \varepsilon}}\frac{V_0}{1 - 2V_0}. \tag{3.3}$$

We note that this relationship between the surface velocity components is universal. It holds not only in the case of quadratic friction since it will remain valid if we replace the velocity module in (1.2) or (2.8) and (2.9) by any function of the latter. For example, for intense winds, a stronger velocity dependence of the surface friction is considered [12]. From (3.3) it is easy to obtain a relationship, also universal, between the module of the surface wind velocity $\left|\mathbf{v}_0\right|$ and its direction

$$\begin{aligned} &\frac{2\left|\mathbf{v}_0\right|}{W} = \sqrt{\frac{\varepsilon}{1 + \varepsilon}}\frac{1}{\sin\psi} + \frac{1}{\cos\psi} = 2\sqrt{\frac{1 + 2\varepsilon}{1 + \varepsilon}}\frac{\cos\left(\psi - \psi_0\right)}{\sin 2\psi}, \\ &\psi_0 = \arctan\sqrt{\frac{1 + \varepsilon}{\varepsilon}}, \end{aligned} \tag{3.4}$$

where the surface wind direction determined by the angle ψ is measured counter-clockwise from the downslope direction.

In the Southern hemisphere, $\psi < 0$ and the first term after the first sign of equality on the right side of (3.4) is nonnegative. Since the left side of (3.4) is nonnegative, from this there follows a restriction on possible surface wind directions

$$|\psi| \ge \arctan\sqrt{\frac{\varepsilon}{1 + \varepsilon}}.$$

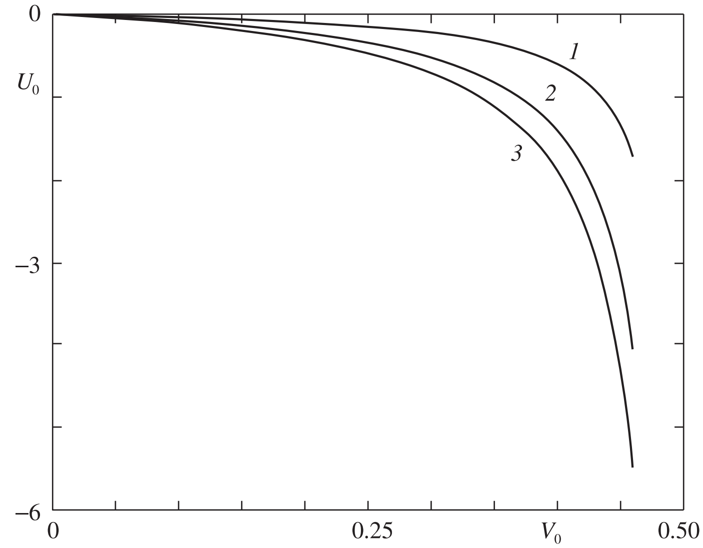

Fig. 2. Relationship between the dimensionless surface velocity components in the Southern hemisphere for $\varepsilon = 0.1$, 1, 10 (*1–3*).

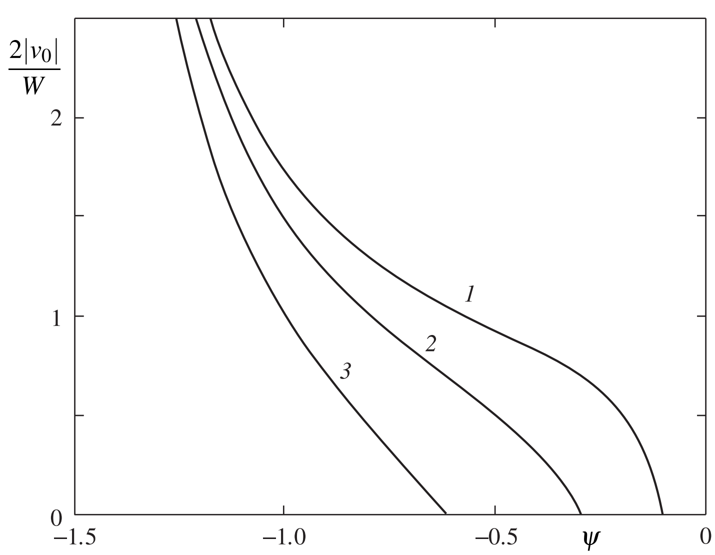

Fig. 3. Relationship between the dimensionless magnitude of the surface velocity and the angle of its deviation from the down-slope direction in the Southern hemisphere for $\varepsilon = 0.01$, 0.1, 1 (*1–3*).

In other words, if we take rotation into account, the flow deviation from the slope direction is bounded from below.

From (2.8) it is easy to obtain another explicit relationship between U_0 and V_0

$$V_0^2 = \left(\frac{A}{U_0}\right)^2 - U_0^2, \quad U_0^2 = \sqrt{A^2 + \frac{V_0^4}{4}} - \frac{V_0^2}{2},$$

whence it follows that inequality (3.2) is satisfied.

The solution depends on several parameters among which the slope steepness, ε, and Q/Q_b may vary most substantially. In many important situations, ε is smaller than unity and may be greater only for very small angles $\varphi < \sqrt{\kappa/\nu}(f/N)$.

High values of the parameter A correspond to moderately intense perturbations of the heat sink on the surface

$$\frac{Q}{Q_b} \ll \frac{\nu}{\kappa}\sqrt{\frac{\varepsilon}{1+\varepsilon}\frac{\sin\varphi}{2c_D}}. \tag{3.5}$$

In this case, as can easily be seen from (2.8), (2.9), and subsequent relations, $|U_0| \approx \sqrt{A} \gg 1$. The dimensionless down-slope velocity component V_0 is close in order to its maximally possible value 0.5 and thus comprises only a small fraction of the transverse velocity. In the limit opposite to (3.5), the velocity components may be of the same order or, depending on ε, the down-slope motion may prevail.

Figure 2 shows a universal relationship between the two surface velocity components that follows from (3.3). In Fig. 3 the relationship between the module and direction of the surface velocity that follows from (3.4) is presented.

We will consider numerical parameters corresponding to the Antarctic conditions.

Let $\rho = 1.3$ kg^3/m, $f = -1.3\times10^{-4}$ s^{-1}, $T_0 = 290$ K, and $\gamma = 2\times10^{-3}$ K/m [9]. Then the Brent–Wyaisyal frequency $N = 0.8\times10^{-2}$ s^{-1}. For the lower layer cooling intensity Q we will use the value 120 W/m^2 presented in [9]. We however note that in [9] there was no vertical dependence and cooling was assumed to be volumetric, whereas in the present study we consider cooling from the surface. For the turbulent transfer coefficient we will take the values $\nu = \kappa = 1$ m^2/s characteristic of the atmosphere. In this case, the vertical background heat flux $Q_b = 2.6$ W/m^2 ($Q/Q_b \approx 50$).

For the littoral Antarctic regions [9] the angle of inclination $\varphi = 5\times10^{-2}$ is typical. In this case, the air flux per unit slope perimeter length is of the order of 10^3 m$^2/s$, $\varepsilon \approx 0.1$, $h \approx \sqrt{2\sqrt{\kappa\nu}/N\sin\varphi} \approx 70$ m, $W \approx 30$ m/s, and $\langle v\rangle \approx 15$ m/s. For $c_D = 10^{-3}$, $A \approx 0.15$ and $B \approx 0.5$. The numerical solution of (2.8), (2.9) yields $V_0 \approx -U_0 \approx 1/3$. The absolute values of both surface velocity components are of the order of 10 m/s and the rotation angle is equal to about $\pi/4$. These velocity values are in general consistent with field data. In [9], for example, for a littoral region the characteristic velocity 12 m/s is presented. In Fig. 4, the dimensionless profiles of the velocity component and the temperature perturbation are shown, the latter normalized in this example to $\gamma hQ/Q_b \approx 7$ K. The slope-ward velocity weakly oscillates and is damped with attitude. Near the surface it is positive, which corresponds to a down-slope cooled air motion. The velocity component transverse to the slope u is everywhere negative in accordance with the sign of the Coriolis acceleration in the Southern hemisphere and decreases with attitude from U_0 to $U_0 - \sqrt{\varepsilon/(1+\varepsilon)}(1-V_0)$. The latter result is universal, as well as the behavior of the temperature perturbation

$$T|_{z=0} = \sqrt{\varepsilon(1+\varepsilon)}\left[U_0 - \sqrt{(1+\varepsilon)/\varepsilon}(1-V_0)\right] < 0,$$

$$T|_{z=\infty} = \sqrt{\varepsilon(1+\varepsilon)}\left[U_0 - \sqrt{\varepsilon/(1+\varepsilon)}(1-V_0)\right] < 0,$$

$$T|_{z=\infty} - T|_{z=0} = 1 - V_0 > 0.$$

For the continental Antarctic regions the angle of inclination $\varphi = 5\times10^{-3}$ is typical [9] and $\varepsilon \approx 10$. In this case, the model yields $\Pi \approx 10^4$ m^2/s, $h \approx \sqrt{2\sqrt{\kappa\nu}/N\sin\varphi\sqrt{\varepsilon}} \approx 120$ m, $\langle v\rangle \approx 90$ m/s; $A \approx 0.05$, $B \approx 0.05$, $U_0 \approx -0.2$, $V_0 \approx 0.15$, $u_0 \approx 35$ m/s, and $v_0 \approx 25$ m/s. Such slope flow velocities are not uncommon in Antarctic. For certain parameter regions, model [9] also yields for continental areas much higher wind velocities than for littoral regions. However, a direct comparison with field data is difficult since we used only tentative values of such parameters as the turbulent transfer coefficients and γ. In the example considered, the velocity values are probably overestimated considerably as compared with the field data. One possible cause lies in a too coarse boundary condition for temperature: at $z = 0$ we fix a heat flux independent of the air temperature. In the latter example, it turns out that air is cooled very intensely. Nevertheless, in accordance with the above-mentioned boundary condition, it continues to give heat to the underlying surface. In the lumped parameter model [9], in a similar situation an attempt was made to “correct” the problem formulation: the given heat sink was assumed to depend on the air temperature. When this temperature

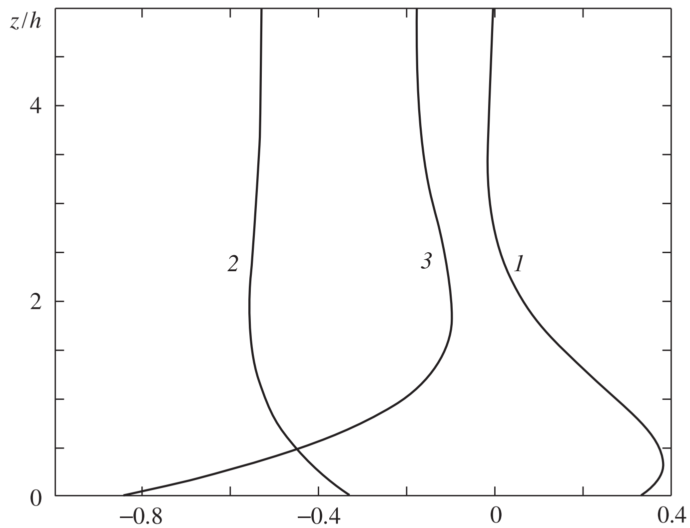

Fig. 4. Example of the dimensionless vertical profiles of the velocity components along (*1*) and perpendicular to (*2*) the slope and the temperature perturbation (*3*).

becomes sufficiently low, the heat sink vanishes. Therefore, a further cooling and slope flow intensification stop, which makes it possible to obtain more realistic results, including in the case of very small angles of surface inclination. A similar correction, for example, the transition to a third-order boundary condition for temperature, may also be expedient in the present model, but its physical and geometrical justification is a serious problem.

Summary. The generalization of the classical slope flow model we considered made it possible to obtain novel results, some of them fairly general. Some results are independent of not only the problem parameters but also the friction law. For example, the mass flux along the slope does not change with taking rotation into account or going over from the no-slip conditions to more general boundary conditions, in particular, to the quadratic friction model. The profile of the velocity component transverse to the slope $u(z)$ is similar to the temperature profile with fairly general boundary conditions for velocity. The relationship between the surface wind velocity components is equally general. For moderately small angles of surface inclination, corresponding to the case of littoral Antarctic regions, the model yields realistic values of the wind velocity. For very small angles of inclination $\sim 5 \times 10^{-3}$, the model possibly overestimates the temperature perturbation and the velocity values. For small angles of inclination, flows may occupy thick medium layers and the assumptions made that the process is stationary and the problem parameters are constant rather coarsen the model. It seems that in any case the analysis of this model is useful for understanding the dynamics of the slope flows.

The model can be improved if we waive certain simplifying assumptions used. In particular, the disturbing heat flux on the surface was assumed to be given, whereas it depends on the surface wind velocity and the air temperature. The turbulent transfer coefficients were assumed to be given and constant. In fact, they depend on the flow intensity and the distance from the underlying surface. Thus, the model can be modified in this direction. It can also be easily generalized to include the case of background geostrophic wind.

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